Instability in Fluid Dynamics

UNIBO Ph.D. Program – Mechanics and Advanced Engineering Sciences
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	Data	Orario	Aula
1	03/05/22 - Martedì	09:00-13:00	TEAMS
2	10/05/22 - Martedì	09:00-13:00	TEAMS
3	17/05/22 - Martedì	09:00-13:00	TEAMS
4	24/05/22 - Martedì	09:00-13:00	TEAMS
5	31/05/22 - Martedì	09:00-13:00	TEAMS

Learning outcomes

The course provides an introduction to the concept of instability in fluid dynamics and offers an outline of the procedure employed for the investigation of the threshold condition for emergence of the instability.

Course contents

Introduction to stability analysis in fluid dynamics

The instability in fluid dynamics occurs for a number of reasons. The main, most common, triggers for the onset of instability are the inertia forces, which produce hydrodynamic instabilities, and the buoyancy forces, which produce thermal instabilities when combined with the thermal advection term. The onset of instability yields a vortex flow or a convective cellular flow. Such flows are obtained, for the hydrodynamic and thermal instabilities, when, respectively, the inertia forces and the buoyancy forces are sufficiently intense. From the practical viewpoint, the expression "sufficiently intense" identifies the threshold conditions for the onset of this cellular flow: the so–called critical conditions.

When the focus of the instability analysis is obtaining the critical values of the governing parameters (e.g. Reynolds number or Rayleigh number) then one can employ a modal stability analysis to answer this question.

When one considers a fluid flow, then understanding whether or not the fluid experiences modal convection, is not the only possible interesting information. For this configuration, the raising instability is convected downstream by the basic flow rate. Thus, from the laboratory reference frame, this instability can or cannot be observed as a function of the strength of the basic flow: when the flow is strong enough the instability is not visible by the observer. The absolute instability analysis answers this question: will the observer in the laboratory reference frame actually see the unstable behaviour?

Modal and absolute instability analysis of thermal convection in fluids

The modal and absolute stability analysis is applied to the case of thermal convection in a fluid: the classical Rayleigh-Bénard problem is considered. This problem has been chosen since the solution is, to a large extent, analytical.

Modal and absolute instability analysis of thermal convection in porous media

A brief introduction to heat and mass transfer in fluid saturated porous media is presented. The modal and absolute stability analysis is applied to the case of thermal convection in fluid saturated porous media: the classical Darcy-Bénard problem is considered. The reason for choosing the thermal convection in fluid saturated porous media is the simplicity of the governing equations. This simplicity helps to handle the analysis since, for this sample case, the solution is numerical.

Detailed contents

1. Definition of stability/instability: thermodynamics (triggering) - mechanical systems (Lyapunov's). Description of stability analysis linear/non linear and two basic examples: logistic equation - heat conduction

2. Definition of stability/instability in fluid systems. Comparison between hydrodynamic and convective instabilities (stability of Poiseuille flow vs Rayleigh-Bénard problem): qualitative description of the governing equations and terms triggering instability.

3. Convective thermal instability: modal vs absolute (distributed vs local). Mathematical foundations of the stability analysis

4. Stability analysis example 1: classical Rayleigh-Bénard problem (free boundaries). Modal and absolute instability analysis carried out *analytically*

5. Stability analysis example 2: classical Darcy-Bénard problem (isoflux boundary conditions). Modal and absolute instability analysis carried out *numerically*