Game Theory, Behavior and The Law - I
A brief introduction to game theory. Rules of the game and equilibrium concepts. Behavioral Games: Ultimatum and Dictator Games. Entitlement and Framing effects.

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Outline

What is Game Theory?

- Considers environments with strategic interaction:
  - i.e. environments in which the payoff of a(t least one) player depends on her/his own decision but also on other players' decisions.
- **This implies that when a players chooses her/his action she/he must form her/his beliefs of other players' actions and how her/his decision is going to affect other players' decisions.**

Game Theory v. Decision Theory

- **Game Theory:**
  - Strategic Interaction
- **Decision theory:**
  - The problem of optimal decision of an individual with no strategic interaction
- **In some environments, GT or DT depends on the specific case:**
  - Example: behaving at the restaurant, are you alone or with some friends (splitting the bill? If you do, how do you order?)

Normative and Positive Game Theory

- **Positive**
  - GT tries to explain and predict behavior in real contexts.
  - People learn about how to play, i.e. they learn what is a rational way of playing (what is best for them), period after period.
  - Although they may temporarily depart from the given definition of ”rationality”, they may tend to it and, also, mistakes by many players may cancel out in the aggregate.
  - This positive perspective is very ambitious: departures from rationality are many and unexpected.
- **Normative**
  - GT investigates what a rational player should do.
  - What does rationality mean? In short: choosing the ”best” option, the ”best response”.
- Somehow recently: new tools for positive analysis (but also for normative):
  - Experimental economics.
  - Behavioral economics tries to model non-rational behavior.

Behavioral Game Theory - An Alternative Approach

- Behavioral game theory starts from the observation that not every player behaves rationally in complex situations.
  - Not all players form beliefs based on an analysis of what others might do (violation of strategic thinking);
  - not all players choose the best response given their beliefs, or at least it is not clear how they define their best response.
- The predictions of traditional game theory are sharp.
- It is not difficult to spot deviations in behavior.
- Deviations are mainly documented by experimental literature.
- Then models are presented, often specialized to the game at hand, and theories are formulated, aiming to explain deviations.
- The difficult part is to formulate a theory that is general enough to encompass different types of deviations. Such general theory is still missing.

How To Deal With Deviations From Rational Behavior

- Focus on specific games.
- Analyze the deviations from equilibria predicted by the traditional theories and present the most important theories put forward to explain such deviations.

Setting Up the Stage

- First we need to learn how to describe and define a game
- Then we will learn how to "solve" it (positive and normative perspectives)
- To define a game we need (informally):
  1. The players
  2. The rules of the game
  3. The outcomes (for each set of actions taken by players)
  4. The payoffs (preferences over outcomes)
- Two descriptions of games: normal (or strategic) form and extensive form representations

Games in Normal Form

- With few players in some special (mostly pedagogical) cases, normal form games can be represented with matrices
- The prisoners' dilemma game

1 \ 2	C	D
C	4, 4	0, 5
D	5, 0	1, 1

Games in Extensive Form

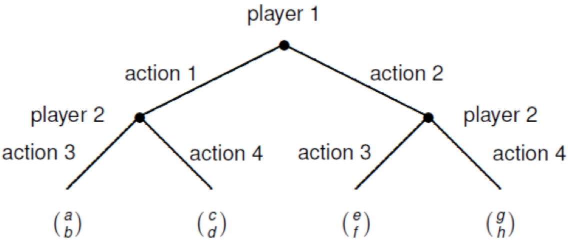
- In a dynamic game, players may act sequentially, they may act several times, and their information may depend on what happened in the past.
- When we need to describe the dynamic in a game, we need another representation that allows us to answer the following questions:
  - Who plays when?
  - What can players do?
  - What do they know?
  - What are their payoffs?

Games in Extensive Form II

- The extensive form of a game describes:
  1. the set of players;
  2. at which point in time which player is called to move;
  3. which actions are feasible for the players when they are called to move;
  4. what players know about the previous history of the game when it is their turn to move;
  5. the payoff of each player as a function of all the possible final histories of the game.

Example of an Extensive Form

- In many cases the extensive form of a game can be represented nicely by a game tree.



Equilibrium and Solution Concepts

- We aim to to find **solutions** of the game that allow us to understand how rational players would act.
- The first step will be simply to rely on **rationality and common knowledge of it**.
  - ...with the concepts of **dominance** and **rationalizability**.
- Then we introduce equilibrium concepts to predict of haw players play.
- Equilibrium requirements include also the assumption that players have (and this is common knowledge) mutually correct expectations.

Dominant and Dominated Strategies

- **(Strictly) dominated strategy (DS)**

A strategy $\hat{s}_i$ of player i is strictly dominated if there exists another strategy $\tilde{s}_i \in S_i$ such that $\tilde{s}_i$ yields a strictly greater payoff than $\hat{s}_i$ no matter what strategies are chosen by his opponents:

$$u_i(\hat{s}_i, s_{-i}) < u_i(\tilde{s}_i, s_{-i}) \quad \forall s_{-i} \in S_{-i} .$$

- **Strictly Dominant strategy**

Strategy s_i^* of player i is a dominant strategy if it strictly dominates all other strategies of player i, i.e. a dominant strategy is a ***strictly best response*** *against all other strategy vectors s_{-i}:*

$$u_i(s_i^*, s_{-i}) > u_i(s_i, s_{-i}) \quad \forall s_i \in S_i \setminus \{s_i^*\}, s_{-i} \in S_{-i}.$$

Solving a Game Using Dominated and Dominant Strategies

- **Definition**: A game is solvable by iterated (strict) dominance if, for each player i, the set of undominated strategies after a "sufficient" number of iterations contains a unique strategy.
- We use rationality (& common knowledge of it)
- Consider again the prisoners' dilemma game
  - The game is (trivially, since just 1 iteration) solvable with deletion of dominated strategies: (D,D)
- Notice: The outcome of the game is not the best for both players from a cooperative view point!
- How can it be possible that they are not able to cooperate?
- Experimental evidence shows that sometimes (often) individuals play (C,C).

Iterated Deletion of Dominated Strategies

1 \ 2	L	M	R
U	1, 0	1, 2	0, 1
D	0, 3	0, 1	2, 0

- How can we solve this? In steps!
  - R is strictly dominated and can be eliminated:
    - First implication: 2 will never play R
    - Second implication: 1 cannot expect 2 to play R
  - But now, once R is "deleted", D is strictly dominated and can be also deleted:
    - 1 will never play D.
    - Hence: 2 cannot expect 1 to play D.
  - But now L is strictly dominated.
- We reached a unique undominated strategy: It follows that the reasonable profile of strategies (the equilibrium) is (U,M).

Nash Equilibrium

- In many games there are no dominant nor dominated strategies. Which strategies should we expect?
- Nash: strategies should form an "equilibrium":
  - each player should have a belief on other players strategies such that his choice is a best response to these beliefs AND these beliefs turn out to be correct (mutually consistent)
- such consistency implies that nobody has an incentive to deviate from this strategy

A Formal Definition

Definition

Nash Equilibrium. The strategy profile $s^* = (s_i^*, s_{-i}^*)$ is a Nash Equilibrium (NE) if each player's strategy is a best response to the other players' strategies (to the expectation thereof). That is, no player has incentive to deviate, if no other player will deviate. (If players find themselves in equilibrium, there is no reason to move away.)

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i^{'}, s_{-i}^*) \; \forall s_i^{'} \neq s_i^*$$

Definition

Best Response. A strategy $s_i^{'}$ is a best response (BR) for player i to the other players' strategies s_{-i} if

$$u_i(s_i^{'}, s_{-i}) \geq u_i(s_i, s_{-i}) \; \forall s_i \neq s_i^{'}$$

How to Find a Nash Equilibrium

- In a two-player game with a finite number of strategies it is easy to find all Nash equilibriums:
  - Simply check all strategy profiles (”combinations”)
  - If strategy spaces are continuous, look at the first order conditions in order to find ”best response functions”. (Also check the second order conditions)

Example: The prisoners' dilemma

- "Two men are arrested, but the police does not possess enough information to keep them in jail. The two men are kept in isolation, the police offers both a similar deal if one testifies against his partner (defects), and the other stays quiet (cooperates), the betrayer goes free (5) and the cooperator receives the full one-year sentence (0). If both remain silent, both are sentenced to only one month in jail for a minor charge (4). If each 'rats out' the other, each receives a three-month sentence (1). Each prisoner must choose to either betray or remain silent"

- **Players.** Prisoners 1 and 2 (row and column respectively).
- **Strategies.** Each player can choose either cooperate (C) or defect (D).
- **Payoffs.** Represent the payoffs in the strategic-form matrix:

1 \ 2	C	D
C	4, 4	0, 5
D	5, 0	1, 1

- There is one Nash equilibrium at {D,D}
- It is also obtained with IEDS method
- They could do better

Example: Stag Hunt game

- "Two hunters simultaneously choose to hunt for rabbits, or to hunt for a stag. Successfully capturing a stag requires coordination, but there will be lots of meat. Anyone can catch a rabbit, but there will be less meat, especially when both are hunting rabbits (all that noise. . .)"
- **Players.** Hunters 1 and 2 (row and column respectively).
- **Strategies.** Each player can choose either Rabbit (R) or Stag (S).
- **Payoffs.** Represent the payoffs in the strategic-form matrix:

	R	S
R	$\underline{3}$, $\underline{3}$	4, 0
S	0, 4	$\underline{5}$, $\underline{5}$

- There are Nash equilibria at {R,R} and {S, S}. Which one gets played? A **coordination issue**
- The latter is Pareto optimal. . . but is the former, somehow, "safer"?

Example: Matching Pennies

- "Two players (row and column) each have a coin. They must simultaneously choose whether to put their coins down heads-up (H) or tails-up (T). If the coins match, row gives column their coin, if they don't match, column gives row their coin."
- **Players.** As usual, row player and column player.
- **Strategies.** Row chooses from {H, T}, and column from {H, T}.
- **Payoffs.** Represent the payoffs in the strategic-form matrix:

	H	T
H	-1, $\underline{1}$	$\underline{1}$, -1
T	$\underline{1}$, -1	-1, $\underline{1}$

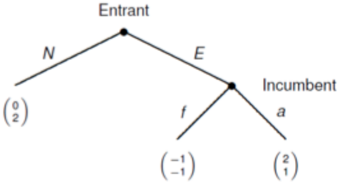
- Both H and T can be best responses, depending on the other player's strategy.
- There is no pure-strategy Nash equilibrium to this game. There is a mixed equilibrium, can you find it? We will see...

Dynamic Games

- Many strategic contexts are dynamic
- This per-se does not necessarily require a new kind of analysis, in fact
  - We can transform almost any dynamic game into its normal form representation
  - And then use the equilibrium concepts analyzed so far for static games
- However there is a problem with this:
  - In dynamic games this procedure may lead to equilibria that do not make very much sense and the issue is the one of CREDIBILITY of a player's strategy
  - We then need to REFINE the equilibrium set, eliminating those unreasonable equilibria
- This is what makes the analysis of dynamic games different!

Credibility: an example, the entry game

- The entrant decides whether to enter (E) the market or not (N)
- If there is entry the incumbent decides whether to fight (f) or to accomodate entry(a).



- The normal form representation is

E \ I	f	a
N	0, 2	0, 2
E	-1, -1	2, 1

- The analysis of the normal form shows that there are two Nash equilibria: (E; a) and (N; f)
  - What is the problem? It is with equilibrium (N;f) why?

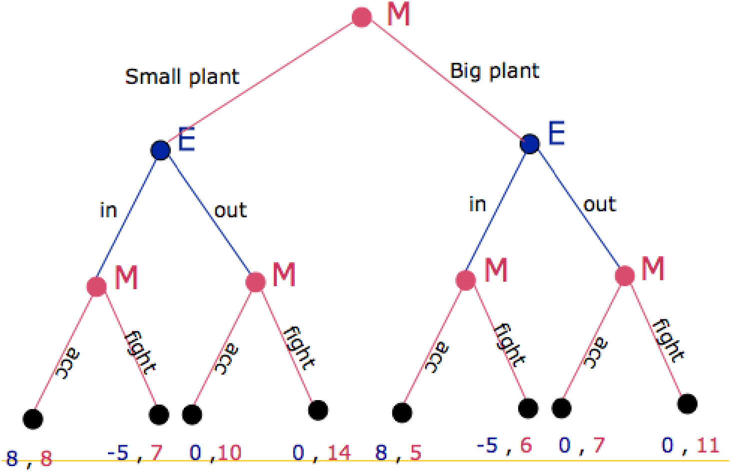
Dynamic Games and Sequential Rationality

- Another way to see the problem with NE (N;f) in the Entry game:
  - actions at decision nodes not reached by play of the equilibrium strategies (here, firm I's action at the decision node following firm E's not chosen E) do not affect firm I's payoff
- As a result, firm I can "announce" or plan to play whatever strategy after N
  - The problem is that what firm I's strategy "says" it will do at the unreached node can actually insure that firm E, taking firm I's strategy as given, wants to play "out"
- We can **rule out** those **non credible announcements** or predictions such as (fight if firm E plays "E") by ALWAYS requiring that wherever a player is called to act, she must play an optimal strategy at that point
- It is the **principle of sequential rationality**: rationality at ANY point in which you are called to take an action, even if it is out of the equilibrium path!
- In the example playing f after E is not sequentially rational for I and then we cannot expect it, we can only expect a after E:
- The NE (N;f) does not satisfy the sequential rationality requirement!

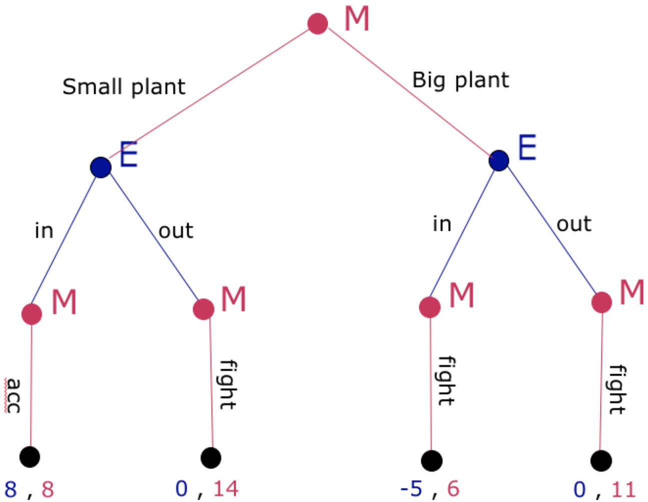
Sequential Rationality in Action

- For the class of finite games with perfect information there is a simple way to apply the requirement of SR, it is the **backward induction principle** or method
- The idea is: first solve for optimal behavior at the "end" of the game (in the entry example: at the post-entry decision node) and then determine what is the optimal behavior at earlier (upward) nodes in the game given the prediction of this later behavior
- Note: after solving optimal (rational) behavior at lower nodes, we can redefine a *reduced game* where we "compact" what will happen at later nodes by anticipating what we have seen is the optimal behavior at those later nodes
- An example...

A Commitment Game: The Big Picture



A Commitment Game: The Big Picture 2



Ultimatum Game

- Bargaining game with Take-it-or-leave-it offers.
  - There is only one round;
  - Sequential moves;
  - A proposer (A) and a receiver (B) bargain about splitting a surplus (say, 10 Euros)
  - Player A makes an offer about splitting such surplus;
  - If player B does not accept, bargaining is over and both players get nothing;
  - Theory predicts that B should rationally accept any offer that A makes (including zero).
  - Experimental evidence contradicts this conclusion.

Ultimatum Game. Experimental Evidence

- Güth,W., R. Schmittberger, And B. Schwarze (1982): "An Experimental Analysis of Ultimatum Bargaining," JEBO.
- According to theory, players should accept any offer they receive, no matter how small
- When proposer and chooser are together in the same room playing the experiment, the most common offer is a 50:50 split
- Also when bargaining is anonymous (through a computer) offers are seldom worse than 75:25
- More precisely (Camerer, 2003):
  - Modal and median ultimatum offers are 40-50% and means are 30-40%.
  - Offers in the very low range 0, 1-10% and in the "hyper-fair" range 51-100% are seldom proposed.
  - Offers of 40-50% are rarely rejected, whereas offers below 20% are rejected half of the times.

Why are Responders "Irrational"?

- Many individuals have preference for being treated fairly.
- For many people, a fair split of $ 10 is close to 50:50 (the concept of fair split may change - but it does not, apparently - when stakes change).
- Rejecting (substantial) sums could be evidence of "negative reciprocity":
  - responders reciprocate unfair behavior by punishing the person who treated them unfairly, even at a cost to themselves (burning money).

Fairness and Negative Reciprocity

- Where do preferences for fairness come from?
  - Evolutionary adaptation: getting angry when treated badly because getting angry had survival value when interacting within the same group of people in a relatively small group (Frank, 1988, *Passions within reason: the strategic role of emotions*).
  - Fairness standards are created by culture, and may differ according to the closeness of kin relations or the degree of anonymity in market exchanges.

A Cross-Cultural Study of the Ultimatum Game

- Henrich et al. 2005. "'Economic Man' in Cross-Cultural Perspective: Behavioral Experiments in 15 Small-scale Societies," Behavioral and Brain Sciences, 28(6), pp. 795-815 provides a formidable cross cultural study.
- Culture seems to matter a lot on how people "play" the ultimatum game.
- Henrich and colleagues did the ultimatum game (and other games) in some simple societies (with Pittsburgh and LA as controls).

Map of Experiments

Figure 1. Locations of the 15 small-scale societies.

Results

Table 2. *Ultimatum game experiment summary statistics*

Group	Mean	No. of Pairs	Percentage female	Stake	Mode (% of sample)[1]	Rejections	Low Rejections[2]
Lamalera[3]	0.57	19	55	10	0.50 (63%)	4/20 (sham)[4]	3/8 (sham)[4]
Aché	0.48	51	54	1	0.40 (22%)	0/51	0/2
Shona (resettled)	0.45	86	45	1	0.50 (69%)	6/86	4/7
Shona (all)	0.44	117	46	1	0.50 (65%)	9/118	6/13
Orma	0.44	56	38	1	0.50 (54%)	2/56	0/0
Au	0.43	30	48	1.4	0.3 (33%)	8/30	1/1
Achuar	0.43	14	50	1	0.50 (36%)	2/15[5]	1/3
Sangu (herders)	0.42	20	50	1	0.50 (40%)	1/20	1/1
Sangu (farmers)	0.41	20	50	1	0.50 (35%)	5/20	1/1
Sangu	0.41	40	50	1	0.50 (38%)	6/40	2/2
Shona (unresettled)	0.41	31	48	1	0.50 (55%)	3/31	2/6
Hadza (big camp)	0.40	26	50	3	0.50 (35%)	5/26	4/5
Gnau	0.38	25	46	1.4	0.4 (32%)	10/25	3/6
Tsimane	0.37	70	51	1.2	0.5/0.3 (44%)	0/70	0/5
Kazakh	0.36	10	45	8	0.38 (50%)	0/10	0/1
Torguud	0.35	10	50	8	0.25 (30%)	1/10	0/0
Mapuche	0.34	31	13	1	0.50/0.33 (42%)	2/31	2/12
Hadza (all camps)	0.33	55	50	3	0.20/0.50 (47%)	13/55	9/21
Hadza (small camp)	0.27	29	51	3	0.20 (38%)	8/29	5/16
Quichua	0.25	15	48	1	0.25 (47%)	0/14[5]	0/3
Machiguenga	0.26	21	19	2.3	0.15/0.25 (72%)	1	1/10

Discussion

- They showed that some of these societies were playing remarkably close to the self-interested player, with very little offers and very low rejection rate.
- Some others were characterized by "hyperfair" offers, with a curious interpretation (insulting competitive gift-giving).
- Generally, societies with lower offers are socially disconnected, with a very low rate of anonymous transactions in the village.
- Seemingly, average offers are positively correlated to the degree of market development and anonymous interaction.
- The more socially connected societies, the more likely that they develop social sharing norms and a fairness standard.

Framing Effects in Ultimatum Games. Hoffman et al. (1994)

- It seems that the way in which strategies are described exerts a strong influence on the way people behave in the ultimatum game.
  - In one study, the ultimatum game is described as an exchange: a seller setting a price for a good that the buyer can take or leave (Hoffman, McCabe, Shachat and Smith (1994), Preferences, Property Rights and Anonomity in Bargaining Games, Games and Economic Behavior, 7: 346-80).
  - Rejection rates are unchanged, but offers decrease by almost 10%.

Instructions. Hoffman et al. (1994)

ULTIMATUM, BUY–SELL, RANDOM ENTITLEMENTS

Instructions

In this experiment you have been paired anonymously with another person. One of you will be the seller, the other the buyer. The seller chooses the selling PRICE. Then the seller's choice is presented to the buyer who chooses to BUY or NOT BUY. In the following table each cell shows the possible profit, in dollars, in the upper right corner for the seller, and in the lower left corner for the buyer. For example, if the seller chooses PRICE = \$8, and then the buyer chooses BUY, the seller will be paid \$8 and the buyer will be paid \$2. If the seller chooses PRICE = \$1, and then the buyer chooses BUY, the seller makes \$1, and the buyer \$9. If the buyer chooses NOT BUY, each of you will be paid nothing, whatever might have been the seller's choice of PRICE. The seller will be given a choice form. After he/she has circled a PRICE choice, the experimenter will circle this PRICE on the buyer's choice form, and the buyer will choose BUY or NOT BUY.

Framing Effects in Ultimatum Games. Larrick and Blount (1997)

- They frame the ultimatum game as a ”common pool ” or ”resource dilemma” (Larrick and Blount (1997), The claiming effect: Why Players are More Generous in Social Dilemmas than in Ultimatum Games, Journal of Personality and Social Psychology, 72: 810-25).
- A resource dilemma is a sequential game in which players make claims from a fixed common pool of resources and get nothing if their claims add up to more than the pool.
- The first player's claim x from the pool is simply an ”offer” to the second player, who can then claim the remaining $Y - x$ or ”veto” the 1st player's claim by claiming more than $Y - x$, exactly as in a ultimatum game.

Larrick and Blount (1997). Results

- They find that offers in the resource dilemma are more generous (not by much though) and rejection rates lower than in a typical ultimatum game.
- They conclude that framing the game as a common resource dilemma creates a "sense of common ownership that makes offers more generous on both sides".

Entitlement Effects

- Sometimes individuals could feel ”entitled” to a certain share of the surplus.
- It might be an ”endowment effect”: since they have been ”given” the surplus to share, they feel they have the right to keep the higher share.
- It might be a matter of self- perception: they feel somehow more deserving.
- They feel they own the surplus.

How to Generate Entitlement

- A very common way to generate entitlement effects in experiments is to have subjects earn their endowment by performing some task (like answering questions, cutting paper, sealing envelopes, etc.)
- Hoffman et al. (1994) have a treatment in which they allocate the role of proposer to the individual in the pair who answers the higher number of general knowledge questions.
- This lowers offers by 10%.
- However, it has the perverse effect of increasing rejection rates in a statistically significant way.
  - The sense of entitlement generated by performing better in the quiz is not shared by responders (who do not feel more "obliged" to accept lower shares).
  - Apparently, responders form self-serving beliefs about the *legitimacy* of the proposer's entitlement.

The Dictator game

- Participants to the experiment are matched in pairs
  - Player A is designated to decide the split
  - Player B is forced to accept whatever split A decides
- This game tests the determinants
  - Fear of rejection
    - Here there cannot be any
  - Sense of fairness
- The result is that a majority of A players chooses to keep no more than 70

Question: Fairness Standards and Social Norm Compliance are Irrational Behavior?

- Rejections in the ultimatum game simply mean that responders are not maximizing their monetary earnings.
- They don't (necessarily) mean that responders are not capable of strategic thinking.
- Responders might get extra (intrinsic) utility from punishing proposers who do not respect fairness standards or violate social norms.
- Proposers might be reluctant to infringe social norms and offer too little for fear of a social sanction (hence, proposers might even be maximizing their monetary earnings avoiding the social sanction).