Introduction to free boundary problems
Introduzione ai problemi a frontiera libera

Proposal Ph.D. Course
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Course disciplinary sector (SSD): MAT/05 - Analisi Matematica

Syllabus

Program: In this series of lectures, we will discuss several free boundary problems arising in the applications, including the Bernoulli problem, the obstacle-type problem and the two-phase problem.

In particular, the aim of the course will be to discuss the common technique usually used in the context of free-boundary problem and some of their applications. In all the cases, we will focus on three of the main issues in the regularity theory for free boundary problems:

1. Optimal regularity of the solution.
2. Blow-up analysis on the free boundary.
3. $C^{1,\alpha}$ regularity of the free boundary.

Each of these issues will be presented for a different model problem. During these lecture, we will get a glimpse in some of the main concept in regularity theory of free-boundary problems:

1. Viscosity formulation of free boundary conditions.
2. Monotonicity formula's (Alt-Caffarelli-Friedman, Almgren, Weiss and Monneau's type formula).
3. Improvement of flatness.

Finally, research directions and open problems will be discussed.

Programma: In questa serie di lezioni discuterem in merito a diversi problemi di frontiera libera motivati da diverse applicazioni, tra cui il problema di Bernoulli, il problema dell'ostacolo and il problema a due fasi.

In particolare, lo scopo principale del corso sarà discutere delle tecniche comunemente utilizzate nel contesto di problemi a frontiera libera e delle loro applicazioni. In ogni problema trattato, cercheremo di focalizzarci sui principali aspetti della teoria di regolarità della frontiera:

1. Regolarità ottimale delle soluzioni.
2. Analisi blow-up della frontiera libera.
3. Regolarità $C^{1,\alpha}$ della frontiera libera.

Inoltre, verranno presentati alcuni degli aspetti e delle tecniche principali, tra cui

1. Formulazioni viscose di condizioni sulla frontiera libera.
2. Formule di monotonia (Alt-Caffarelli-Friedman, Almgren, Weiss e Monneau formula).
3. Improvement of flatness.

Tempo permettendo, verranno illustrate alcune possibili linee di ricerca e problemi aperti.

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4. De Silva D., "Free boundary regularity for a problem with right hand side", Interfaces Free Bound. 13 (2011), no. 2, 223–238.

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