Titolo del corso: **Topics in stochastic Analysis: Singular Control and Malliavin Calculus**

Docenti: **Giorgio Ferrari** (University of Bielefeld), **Arturo Kohatsu-Higa** (Ritsumeikan University)

Membri del collegio proponenti: Stefano Pagliarani, Andrea Pascucci

Ore frontali di lezione: **20 (10+10)**

Periodo di lezione: **marzo/giugno 2025**

Settore/i disciplinare del corso: **MAT/06**

Tipologia di corso: **Base**

Modalità di verifica dell'apprendimento: **Esame scritto**

Abstract del corso:

Module 1 (Giorgio Ferrari): In this class we will introduce the theory of singular stochastic controls in a Markovian framework. This class of dynamic optimization problems find natural applications in Economics, Finance, Operations Research, and Engineering. In particular, we will investigate the intimate relation to optimal stopping theory and free-boundary problems, as well as to reflected diffusion processes. Finally, if time allows, we will consider a class of stationary mean-field games with singular controls.

Module 2 (Arturo Kohatsu-Higa): Malliavin calculus, also known as the stochastic calculus of variations, provides tools for analyzing the smoothness of functionals of stochastic processes. It is particularly useful in understanding the regularity properties of solutions to stochastic differential equations (SDEs), including those arising in mathematical finance. For example, Malliavin calculus can be used to study the smoothness of the density, which is crucial for practical applications like numerical simulations and approximations of prices and sensitivities.

Programma del corso:

Module I:

- Motivation of singular stochastic controls via an example

- Formalization of a general class of Markovian singular stochastic control problems in R^n

- Dynamic Programming Principle Equation and Verification Theorem for Markovian singular stochastic control problems in R^n

- The optimal policy in terms of the solution to a Skorokhod reflection problem. Challenges in R^n, n>1, and the optimal solution in the one-dimensional case

- One-dimensional stationary mean-field games with singular controls

Module II:

- Stochastic derivative

- Chaine rule
- Clark-Ocone formula
- Applications to mathematical finance