Titolo del corso: Geometry of tensor decomposition

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Ore frontali di lezione: 20

Periodo di lezione: Gennaio - Febbraio 2025

Settore/i disciplinare del corso: Mat 02/03

Tipologia di corso: Base

Modalità di verifica dell'apprendimento: Esame orale

Abstract del corso: Over the last 60 years tensors and multilinear algebra made their way to the applied sciences and the problem of tensor rank decomposition acquired an increasingly central role. One of the main advantages of working with tensors instead of matrices is that tensors very often admit a unique rank decomposition. Under this perspective, after translating applied problems of different fields in the language of tensors, the uniqueness of the tensor rank decomposition represents a unique way of interpreting latent variables. In this course, we will first introduce tensors and explore different notions of rank as well as different notions of symmetries related to tensors. Then, we will study such concepts from a geometric point of view, introducing algebraic varieties related to tensors, and understanding classical results such as Terracini Lemma and the Alexander-Hirschowitz classification for defective Veronese varieties. Lastly, we will focus on the identifiability problem for tensors and we will derive standard results on identifiability of tensors that can be useful in different contexts of applied sciences.

Programma del corso:
L1: From matrices to tensors, tensor rank
L2: Flattenings and multilinear rank
L3: Concision process and examples of rank computation
L4: Bounding the rank via substitution method and Koszul flattenings
L5: Symmetric tensors and catalecticants
L6: Sylvester's algorithm for binary forms
L7: Secant varieties of tensor related varieties and Terracini's Lemma
L8: A-H classification (defective cases)
L9: Identifiability and Kruskal criterion
L10: Geometric perspective on identifiability