A boost to Higgs Physics: new regimes at high energy

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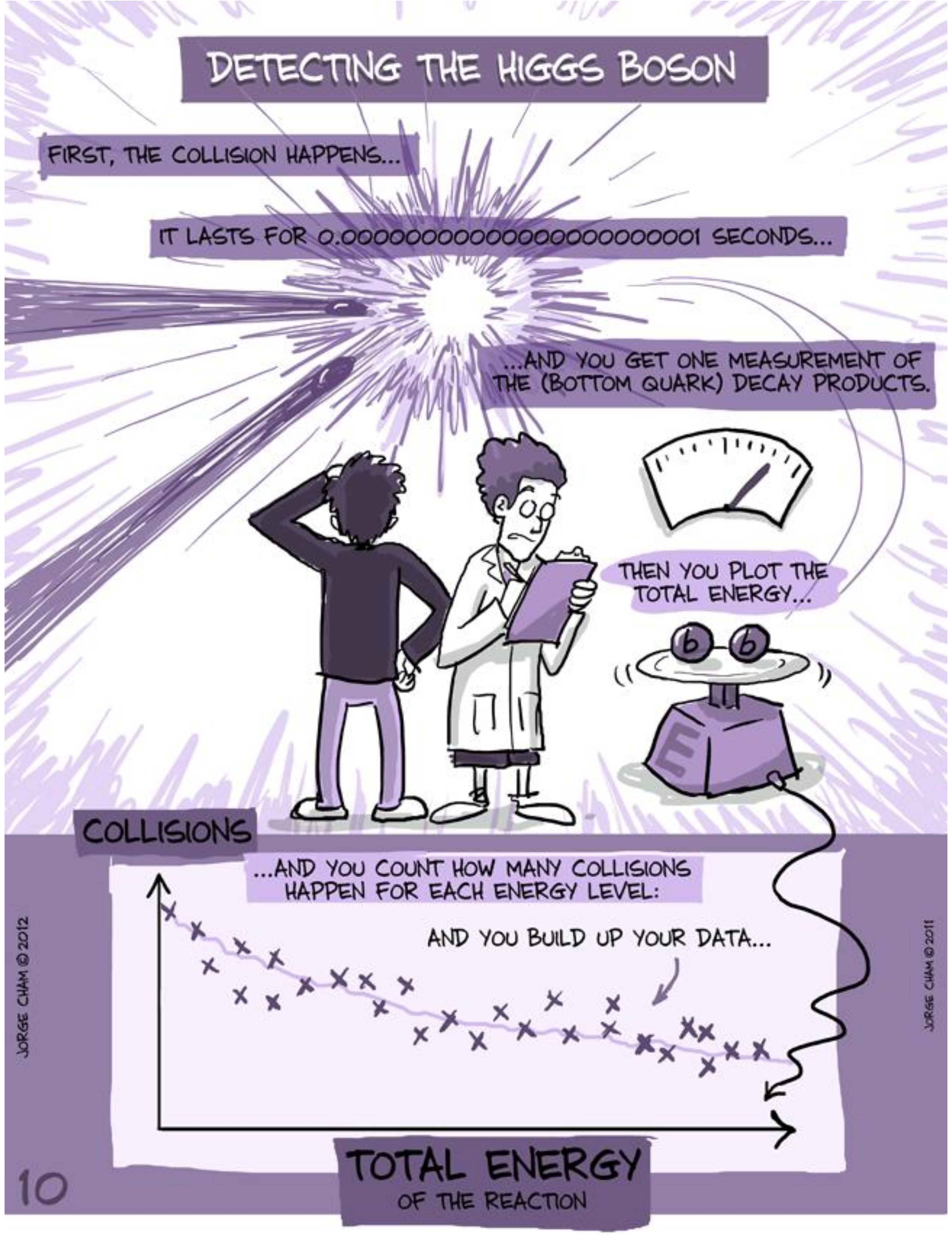
Corso di Dottorato - 2019/2020

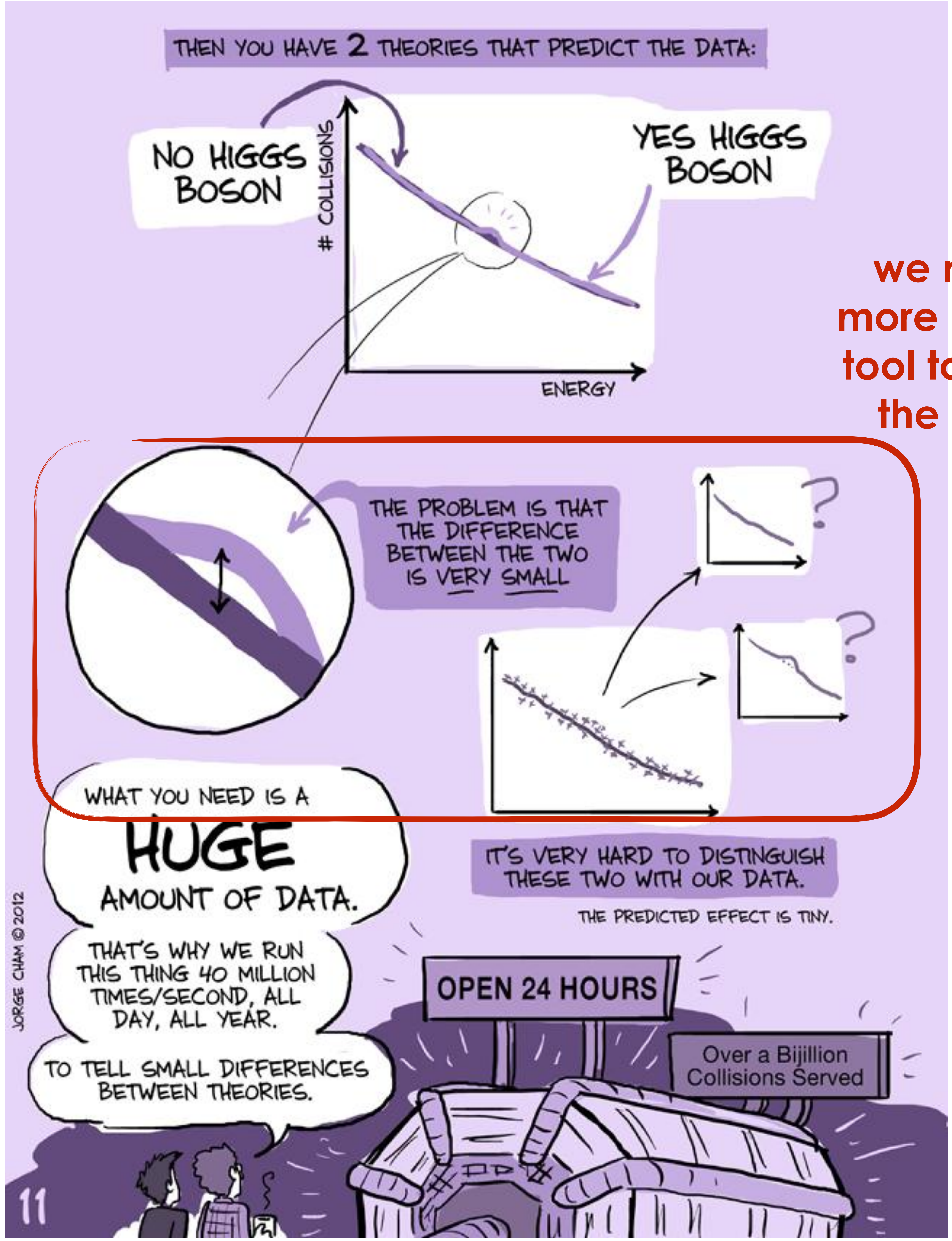
Course outline

- Theory reminder
- Higgs boson production and decay modes
- Higgs boson discovery by ATLAS and CMS
- Higgs boson mass measurement by ATLAS and CMS
- Overview of ATLAS and CMS analyses about Higgs
- Signal/background discrimination techniques
  - boosted regimes
    - tagging, large-radius jets substructure, re-clustering
  - multivariate analysis and deep neural network
- **Signal extraction techniques**
  - **likelihood and test statistic**
  - **CLs method**
- ttH analysis: an example

Signal extraction techniques

http://www.phdcomics.com/





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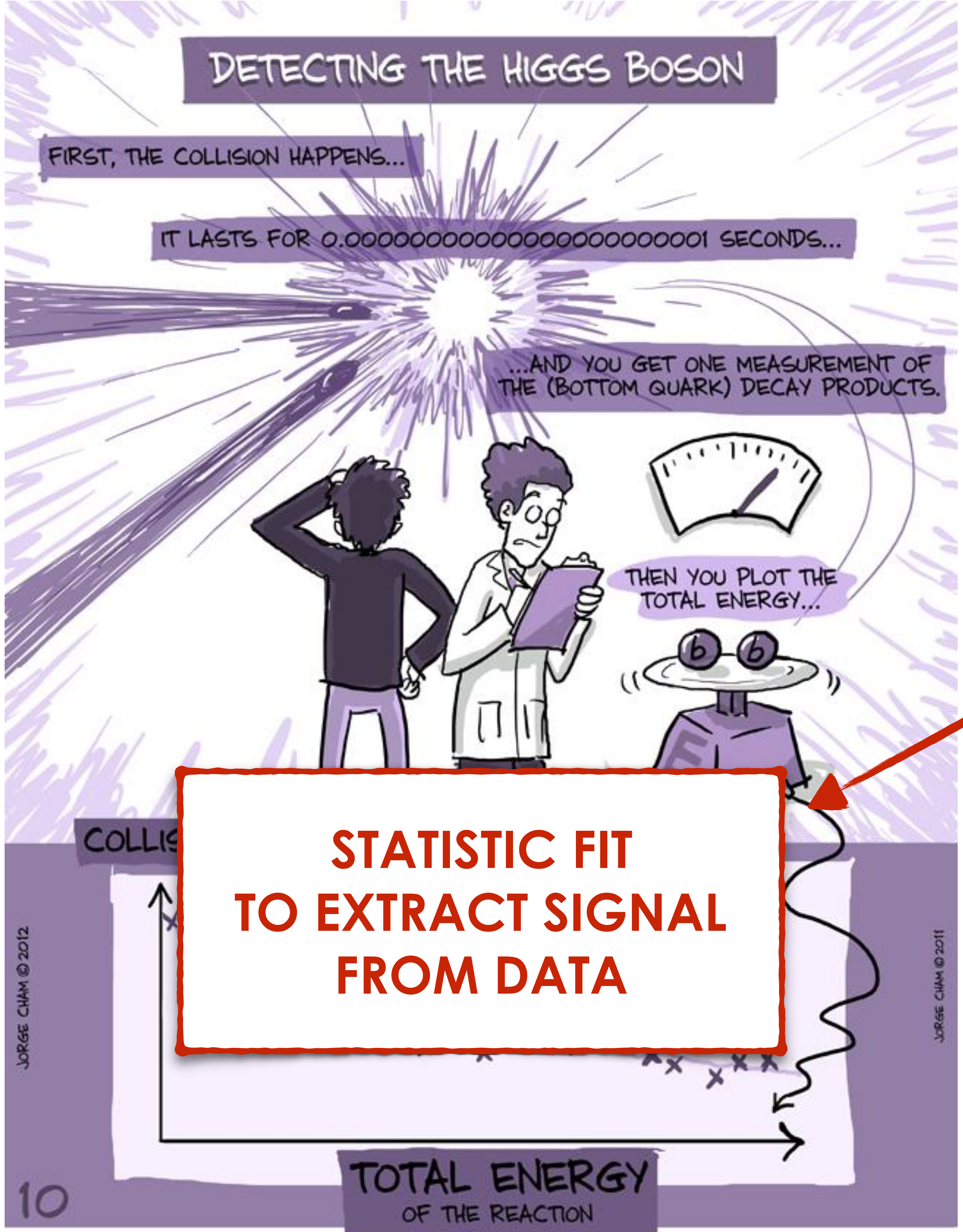
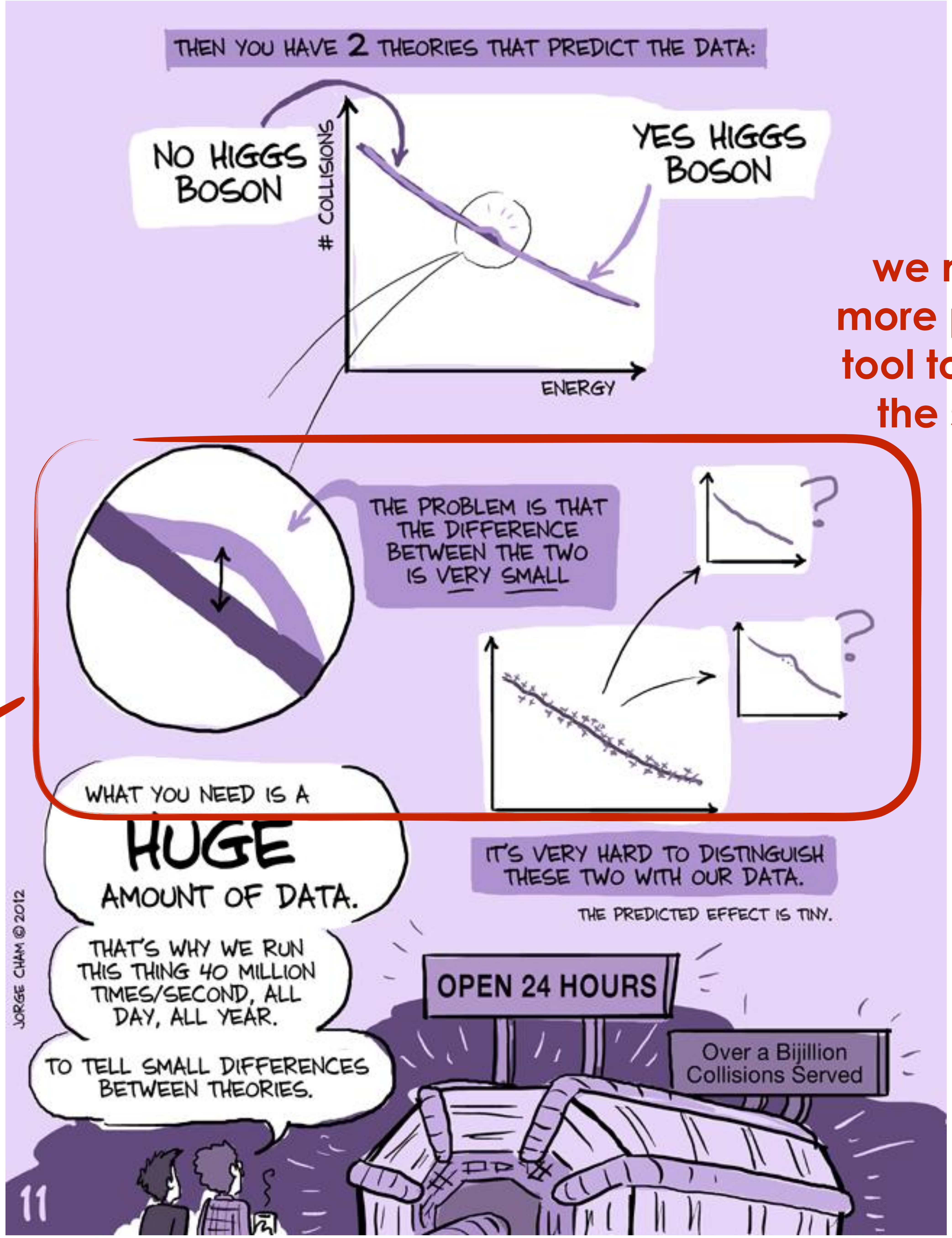


STATISTIC FIT TO EXTRACT SIGNAL FROM DATA



we need a more powerful tool to extract the signal!

Statistical tests

Some notions

- goal of a statistical test is to make a statement about how well the observed data stand in agreement with given predicted probabilities (a **hypothesis**);

- **null hypothesis, H_0**: hypothesis under consideration, specifying a probability density f(x) of a random variable x
  - **simple** if it determines f(x) uniquely
  - **composite** if the pdf is defined but not the values of at least one free parameter θ, f(x,θ).

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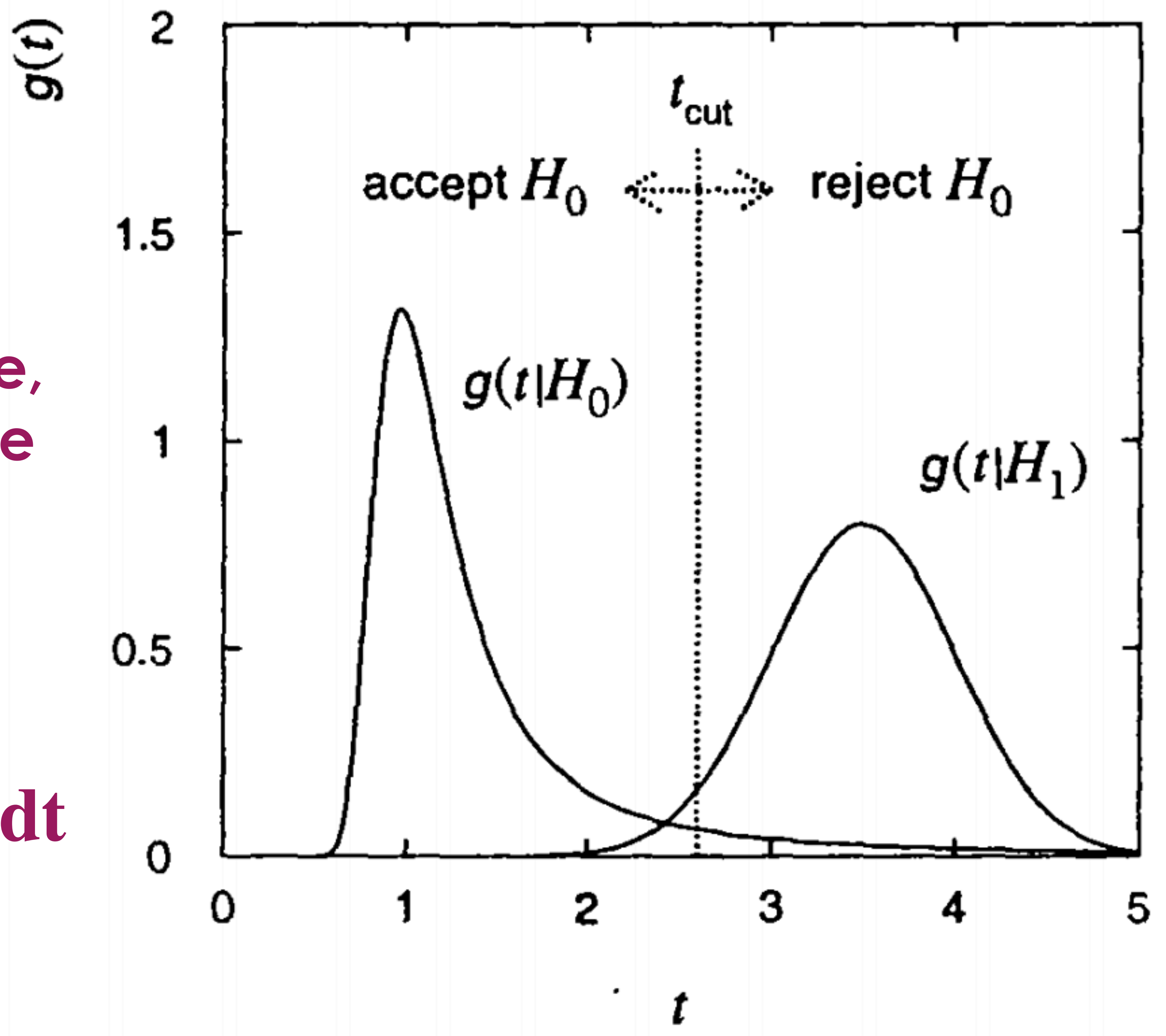
An example

- data: *n* measured values $\mathbf{x} = (x_1, x_2, \ldots, x_n)$;
- set of hypotheses: H_0, H_1; → $f(x|H_0)$, $f(x|H_1)$
- construction of a function of the measured variables called a **test statistic t(x)**, to investigate the agreement between the observed data and a given hypothesis; → $g(t|H_0)$ if H_0 is true, $g(t|H_1)$ if H_1 is true
- statement about the compatibility between the data and the various hypotheses in terms of a decision to accept or reject a given null hypothesis H_0, by defining a **critical region** for t:
  - chosen such that the probability for t to be observed there, under assumption of the hypothesis H_0, is some value a, called the **significance level** of the test; →

$$\alpha = \int_{t_{cut}}^{\infty} g(t|H_0)\,dt$$

- accepting (or not rejecting) the hypothesis H_0, if $t<t_{cut}$.



Statistical tests

How do we chose the critical region?

- depending on the **efficiency and purity of the selected events** desired in the analysis;
- **multidimensional test statistics** chosen and simple hypothesis **H_0** tested, to select events of a given type; **$t = (t_1, \ldots t_m)$**
- simple alternative hypothesis **H_1**;
- **also background events**, so that the signal purity in the selected sample will in general be less than 100%.

- **Neyman-Pearson lemma** states that the acceptance region giving the highest power (and hence the highest signal purity) for a given significance level a (or selection efficiency ε = 1 - a) is the region of t-space such that:

$$\frac{g(t|H_0)}{g(t|H_1)} > c$$

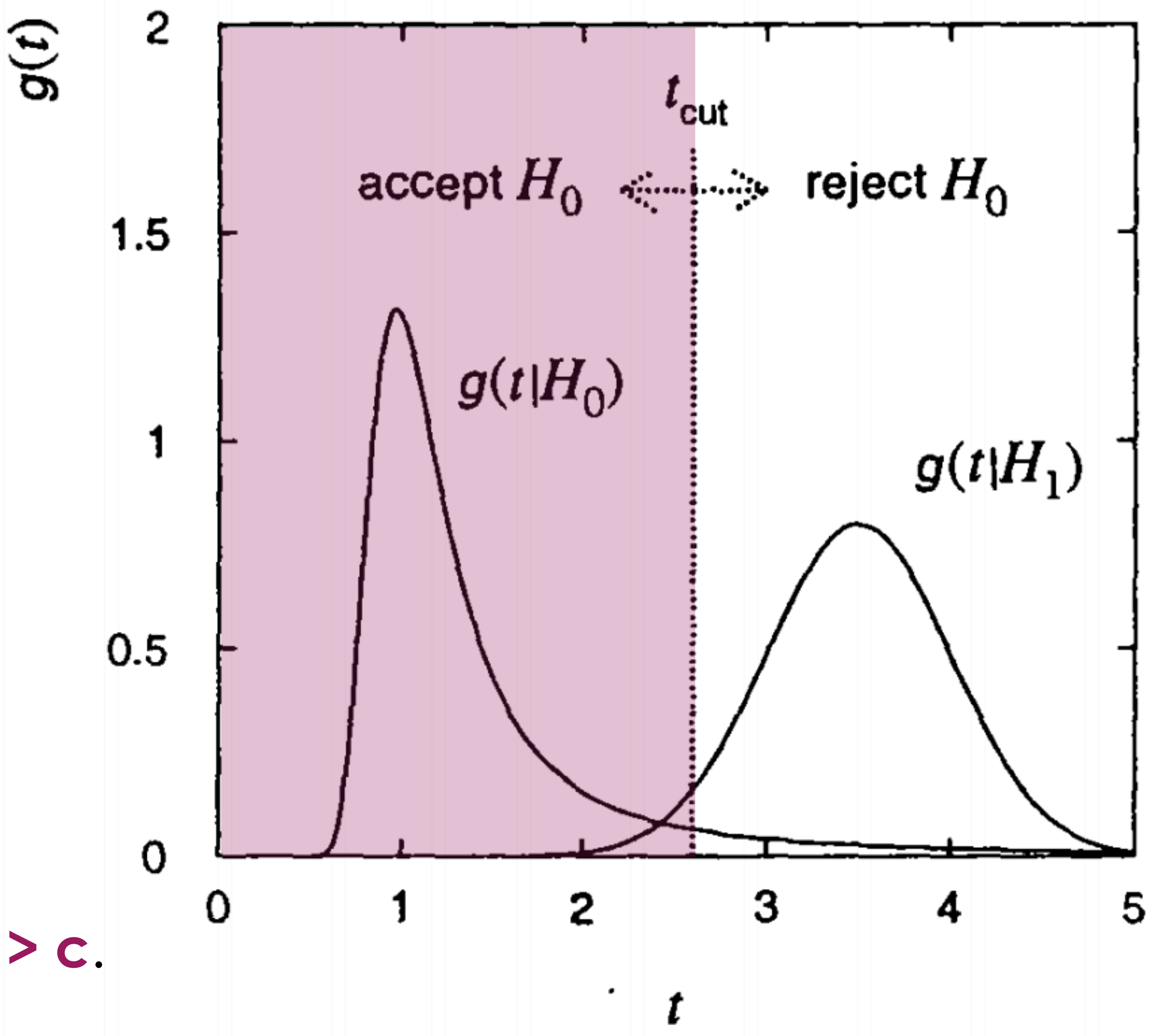
c = constant which is determined by the desired efficiency

equivalent to a test using a one-dimensional statistic given by the ratio

$$r = \frac{g(t|H_0)}{g(t|H_1)}$$

Lilkelihood ratio for simple hypotheses H_0 and H_1

The corresponding acceptance region is given by **r > c**.



Significance of an observed signal

Goodness-of-fit

- to judge whether a discrepancy between data and expectation is sufficiently significant to merit a **claim for a new discovery**;
- given by stating the so-called **P-value**, i.e. the probability P, under assumption of the hypothesis in question H_0, of obtaining a result as compatible or less with H_0 than the one actually observed:
  - also called the **observed significance level** of the test;
  - the fraction of times one would obtain data as compatible with H_0 or less so if the experiment (i.e. 20 coin tosses) were repeated many times under similar circumstances.

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An example

- special type of signal event, the number n_s of which can be treated as a Poisson variable with mean ν_s;
- a certain number of background events n_b, can be treated as a Poisson variable with mean ν_b;
- total number of events found is $\mathbf{n} = \mathbf{n_s} + \mathbf{n_b}$, which is a Poisson variable with mean $\nu = \nu_s + \nu_b$;

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- probability to observe n events:
- from experiment, n_{obs} events found;
- to quantify our degree of confidence in the discovery of a new effect ($\nu_s \neq 0$) we can compute **how likely it is to find n_{obs} events or more from background alone**:

$$f(n;\nu_s,\nu_b) = \frac{(\nu_s+\nu_b)^n}{n!}e^{-(\nu_s+\nu_b)}$$

$$P(n \geq n_{obs}) = \sum_{n=n_{obs}}^{\infty} f(n;\nu_s=0,\nu_b)$$

$$= 1 - \sum_{n=0}^{n_{obs}-1} f(n;\nu_s=0,\nu_b)$$

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- if we expect $\nu_b = 0.5$ background events and we observe $\mathbf{n_{obs} = 5}$, then the P-value is **1.7 x 10^{-4}**;

- this is not the probability of the hypothesis $\nu_s = 0$.

- **It is rather the probability, under the assumption $\nu_s = 0$, of obtaining as many events as observed or more.**

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Danger!

- We assumed ν_b to be known without error;
- if we had $\nu_b = 0.8$ instead, the P-value would increase by almost an order of magnitude to **1.4 x 10^{-3}**!
- **important to quantify the systematic uncertainty in the background when evaluating the significance of a new effect!**

Likelihood-based test

Standard procedure to discover a new signal process

- A widely used procedure to establish discovery (or exclusion) in particle physics is based on a significance test using a **profile likelihood ratio** as a test statistic;

- necessary that the **model predictions** for data distributions **represent accurately the underlying theory being tested**:
  - any errors due to approximations (e.g. in detector modelling) should be negligible in the full parameter space;

- By including **additional parameters to the model** (accounting for systematics effects) it is possible to approach this ideal situation more closely, but resulting in a loss in sensitivity.

H_0 only background
H_1 both background and signal

Likelihood-based test: building the Likelihood

How to build a Likelihood

- **For each event** selected in the signal sample, a variable **x** of a certain kinematic quantity can be measured;
- values **n = (n_1, ..., n_N)** can be used to construct a **histogram of N bins**.

expectation value of n_i

$$\mathbf{E[n_i] = \mu s_i + b_i}$$

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$$s_i = s_{tot} \int_{bin\ i} f_s(x; \theta_s)\, dx \qquad b_i = b_{tot} \int_{bin\ i} f_b(x; \theta_b)\, dx$$

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- subsidiary measurements performed **to help constraining the NPs**;
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provides information on the background normalisation parameter b_{tot}

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calculable quantities depending on the parameters θ

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- likelihood function is defined as the **product of Poisson probabilities for all bins**:

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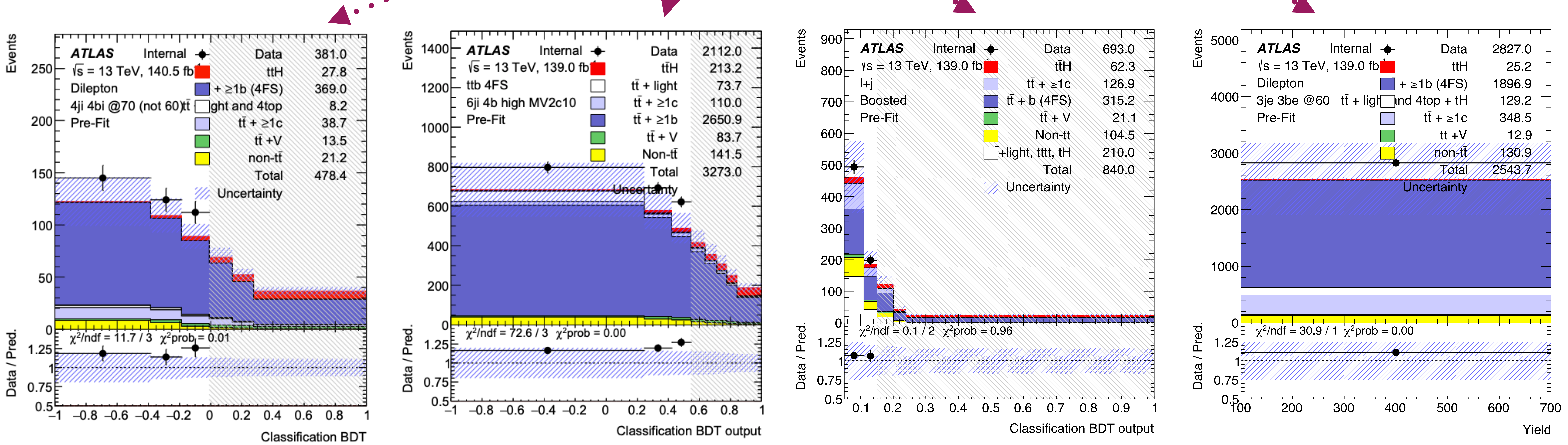

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Profile Likelihood ratio

- To test a hypothesised value of μ:

$$\lambda(\mu) = \frac{\mathscr{L}(\mu, \hat{\hat{\theta}})}{\mathscr{L}(\hat{\mu}, \hat{\theta})}$$

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value of θ

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maximising $\mathscr{L}(\hat{\mu}, \hat{\theta})$ and representing the unconditional maximum-likelihood (ML) estimators

- $\lambda(\mu)$ **assumes values between 0 and 1 (at $\mu = \hat{\mu}$)**;
- λ **close to 1** implies **good agreement between data and the hypothesised value of μ**;
- The presence of the nuisance parameters broadens the profile likelihood as a function of μ
  - **loss of information about μ due to the systematic uncertainties**.

Likelihood-based test: test statistic

To establish an upper limit on the strength parameter μ

- upper limit is obtained by **testing μ against the alternative hypothesis consisting of lower values of μ**;
- definition of a test statistic **q_μ**:

$$q_\mu = \begin{cases} 0, & \mu < \hat{\mu} \\ -2\ln\lambda(\mu), & \mu \geq \hat{\mu}, \end{cases}$$

→ to not represent the values with $\mu < \hat{\mu}$ with less compatibility with respect to the μ obtained from the data

To clarify the formula and the results

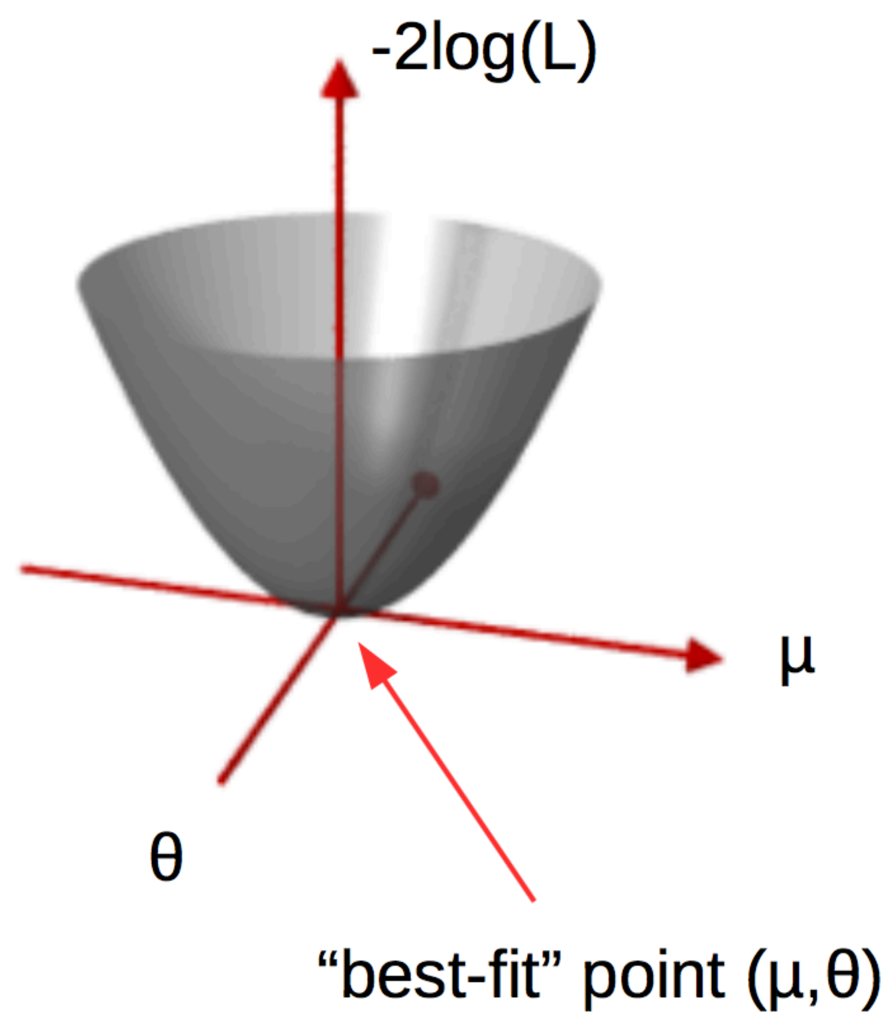
- in the conditions of the central limit theorem, given enough statistics:

$$\lambda(\mu) \approx exp(-\chi^2/2) \quad \rightarrow \quad q_\mu = \chi^2(\mu) \qquad \text{for } \mu > \hat{\mu}$$

high values of q(μ) are equivalent to high values of a χ^2

↓

incompatibility between the data and the test hypothesis



Likelihood-based test: p-value

To quantify the level of agreement between data and hypothesised μ

- definition of p-value, relative to the test statistic;
- for an observed value $\mathbf{q_{\mu,obs}}$:

$$\mathbf{p}_\mu = \int_{\mathbf{q}_{\mu,\mathbf{obs}}}^{\infty} \mathbf{f}(\mathbf{q}_\mu \,|\, \mu)\ \mathbf{dq}_\mu$$

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hypothesis being tested

Likelihood-based test: p-value and significance

To quantify the level of agreement between data and hypothesised μ

- definition of p-value, relative to the test statistic;
- for an observed value $q_{\mu,obs}$:

$$p_\mu = \int_{q_{\mu,obs}}^{\infty} f(q_\mu \mid \mu)\, dq_\mu$$

pdf of q_μ assuming the hypothesis μ

value of μ assumed in the distribution of the data

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Convenient to define also the significance Z

- If x is a Gaussian distributed variable with mean m_x;
- $\hat{x}$ ($\hat{x} > m_x$) is defined as the value of x which has an upper-tail probability equal to the p-value.
- **Z** is defined as the **number of standard deviations of $\hat{x}$ with respect to m_x**:

$$Z = \Phi^{-1}(1 - p)$$

Φ = quantile of the standard Gaussian

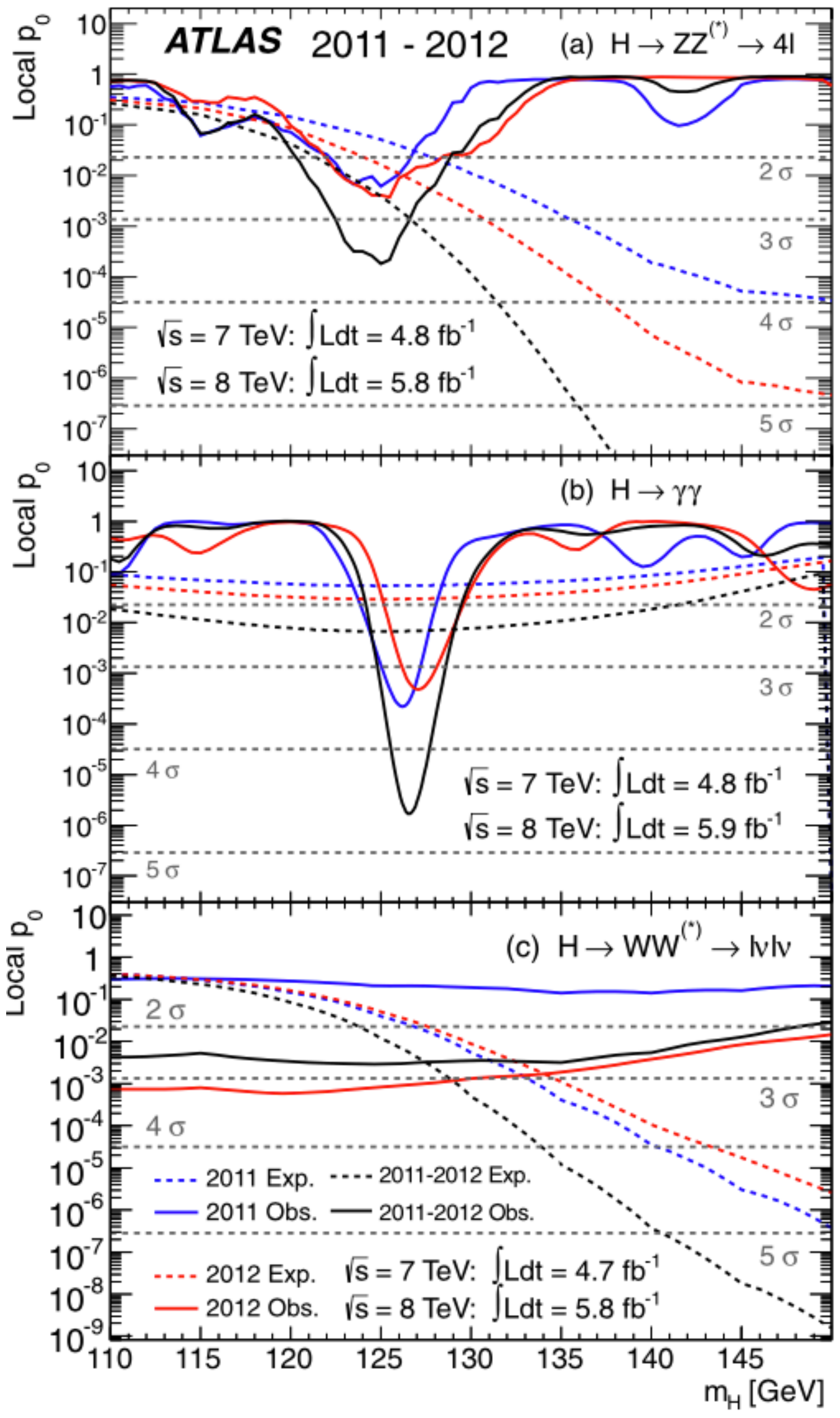
- **For a signal process such as the Higgs boson**, the appropriate level to constitute a physics discovery is a significance of at least **Z = 5**, corresponding to a **p-value = 2.87×10^{-7}**;

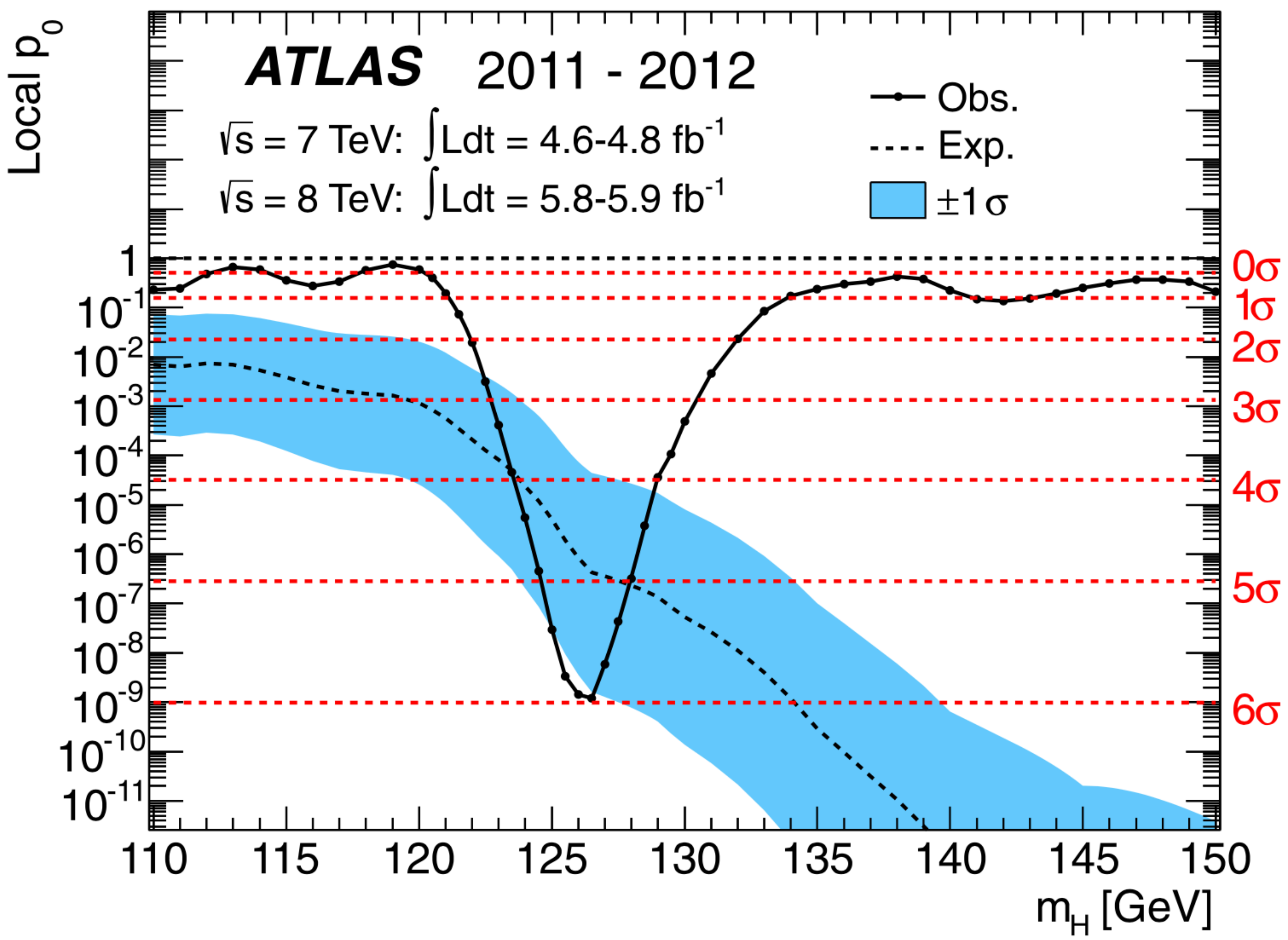
- **for excluding a signal hypothesis**, a threshold p-value of **0.05** (i.e., 95% confidence level, CL) is often used, which corresponds to **Z = 1.64**.

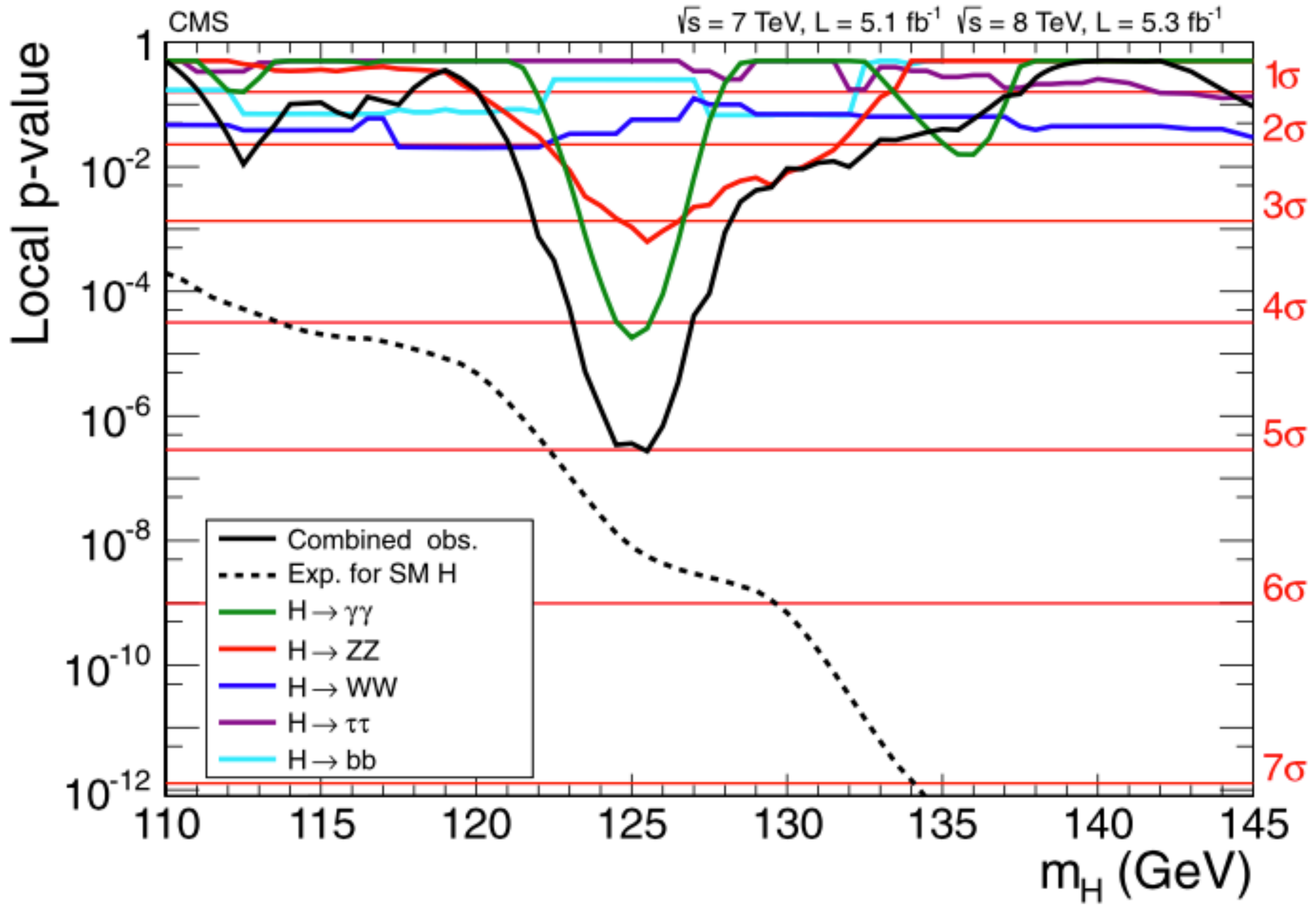
* quantile = inverse of the cumulative distribution

Likelihood-based test: p-value and significance

examples







Sensitivity of an experiment

Asimov data sets

- characterised not only in the significance obtained from a single data set, but rather in the **expected median significance to reject different values of μ**;
- estimator is evaluated by using the so called **“Asimov” data set**, replacing the ensemble of real data with a distribution obtained by a **suitable weighted addition of the MC samples**, according to the chosen test statistic.

For exclusion limit setting

sensitivity characterised by the median significance, assuming data generated using the μ = 0 hypothesis, with which one rejects a nonzero value of μ (usually μ = 1 is of greatest interest)

For discovery

sensitivity characterised by the median, under the assumption of the nominal signal model (μ = 1), with which one would reject the background-only (μ = 0) hypothesis

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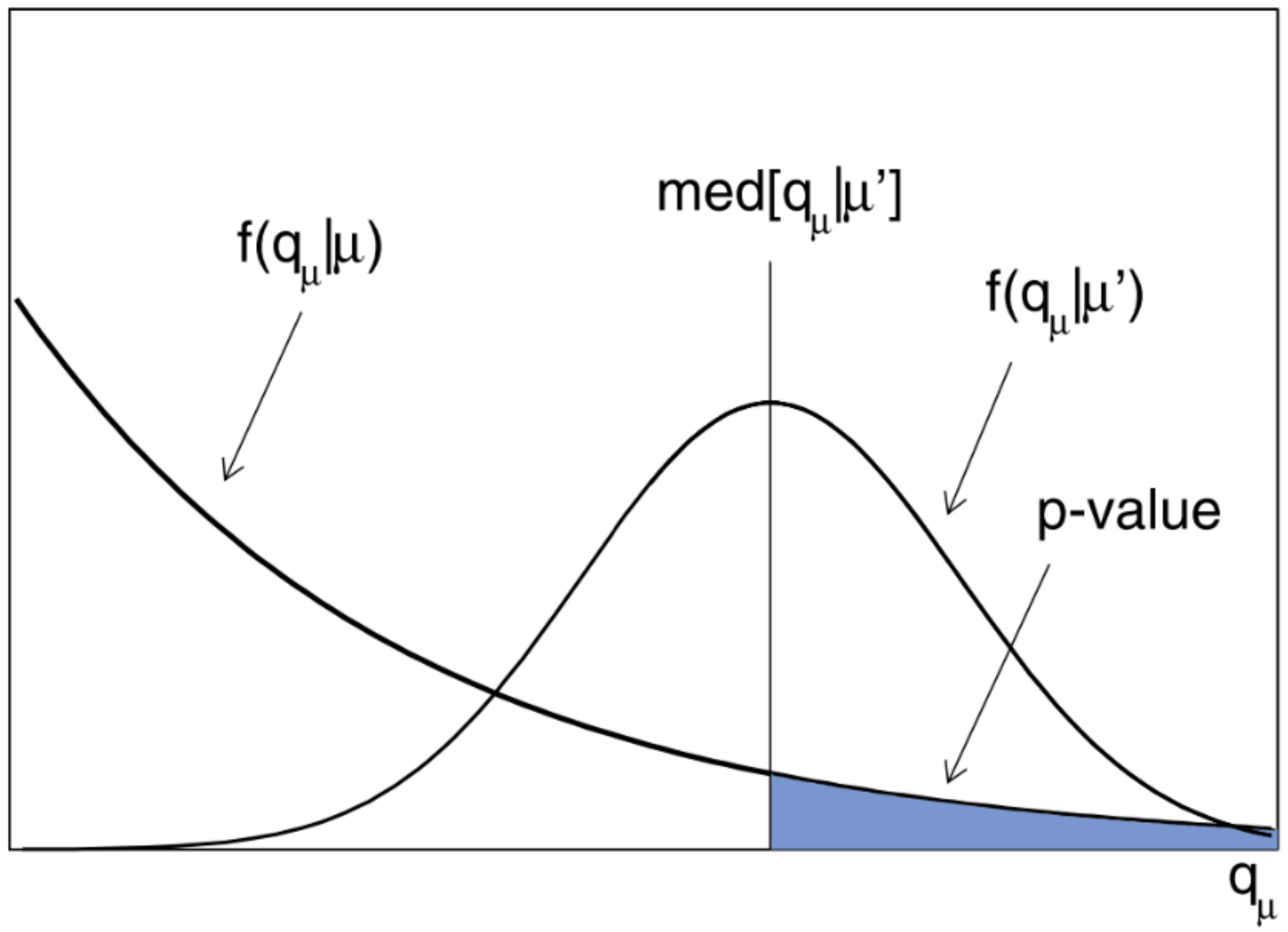

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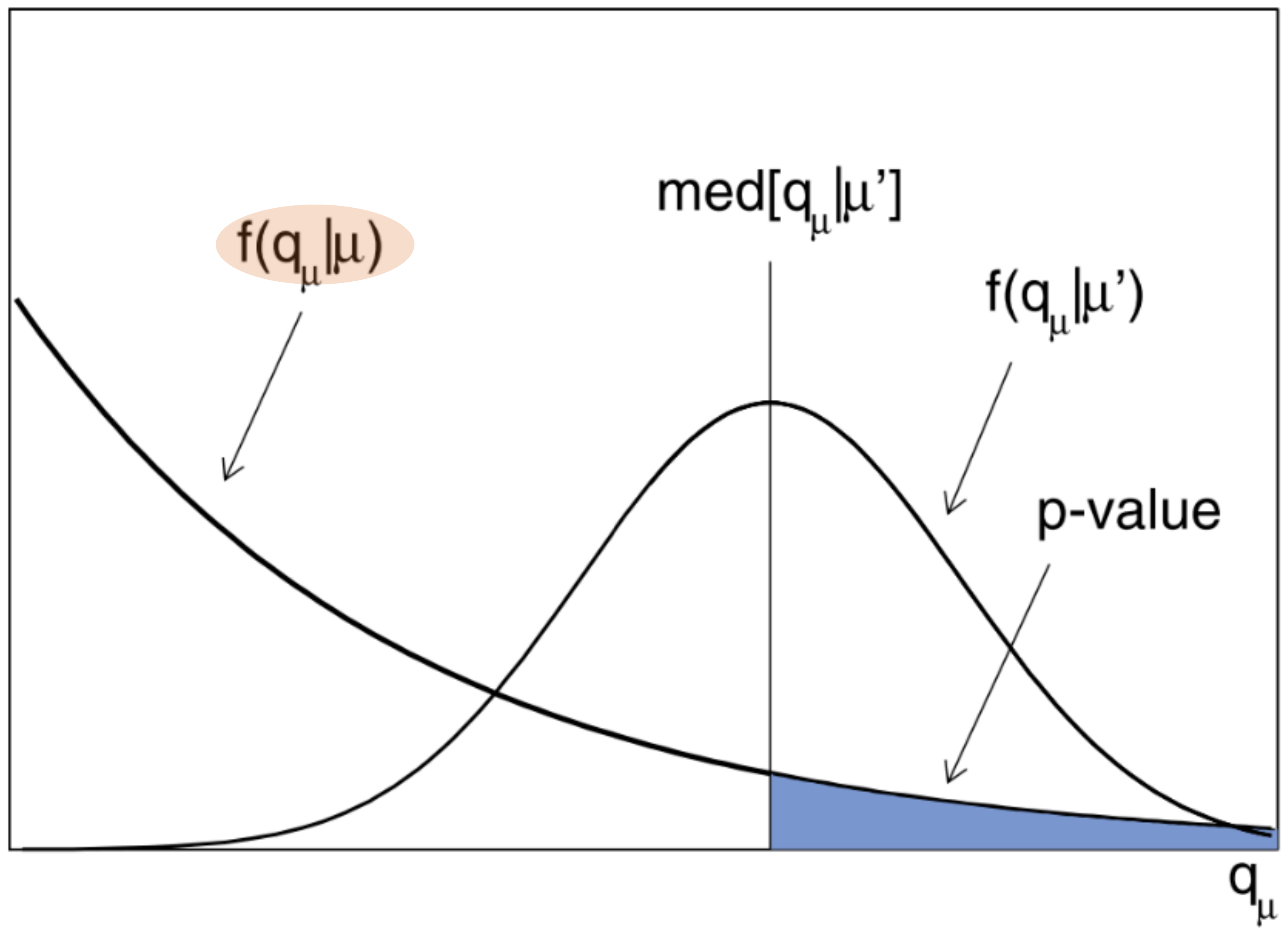

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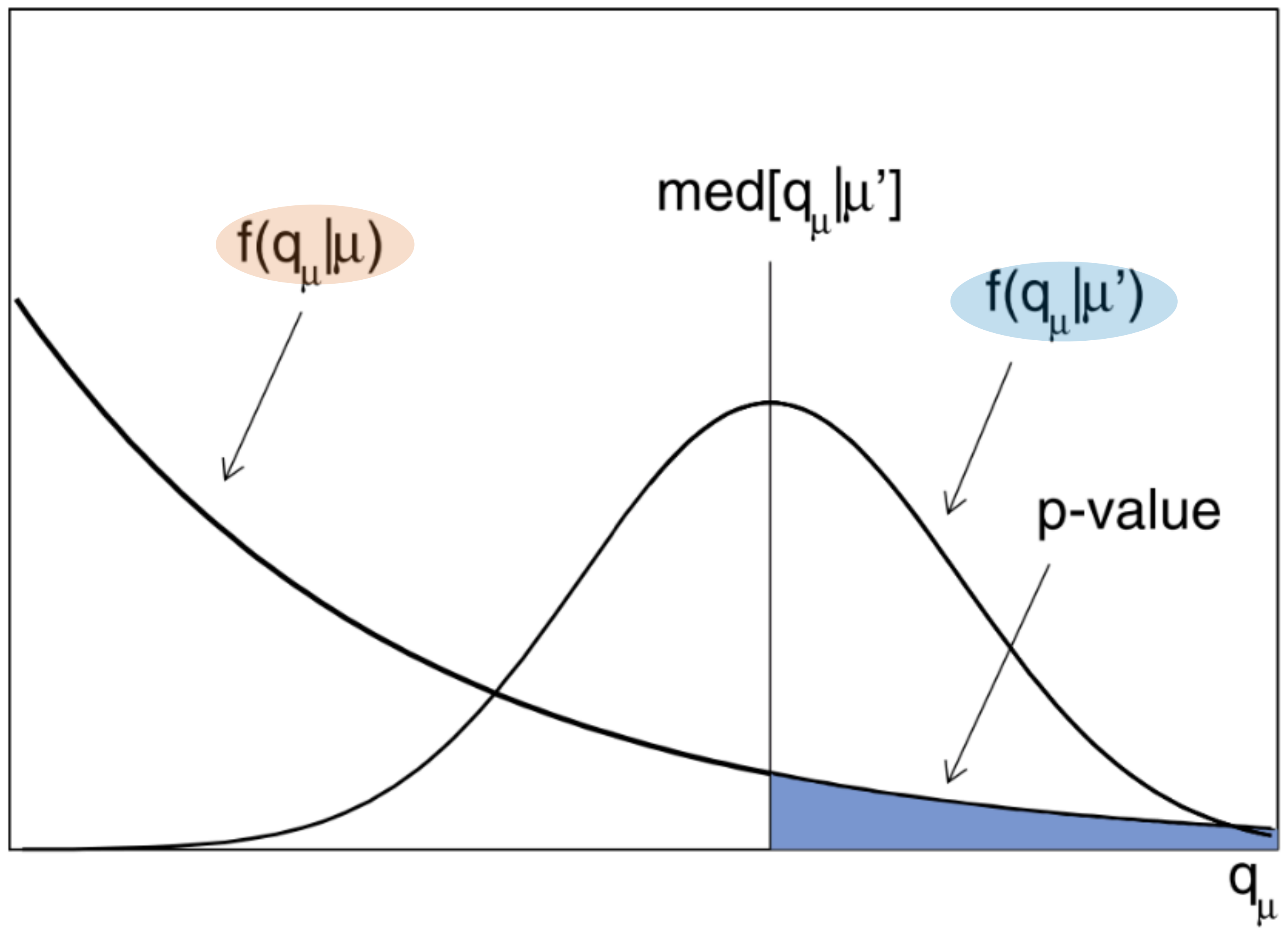

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shifted to higher value of q_μ, corresponding on average to lower p-values

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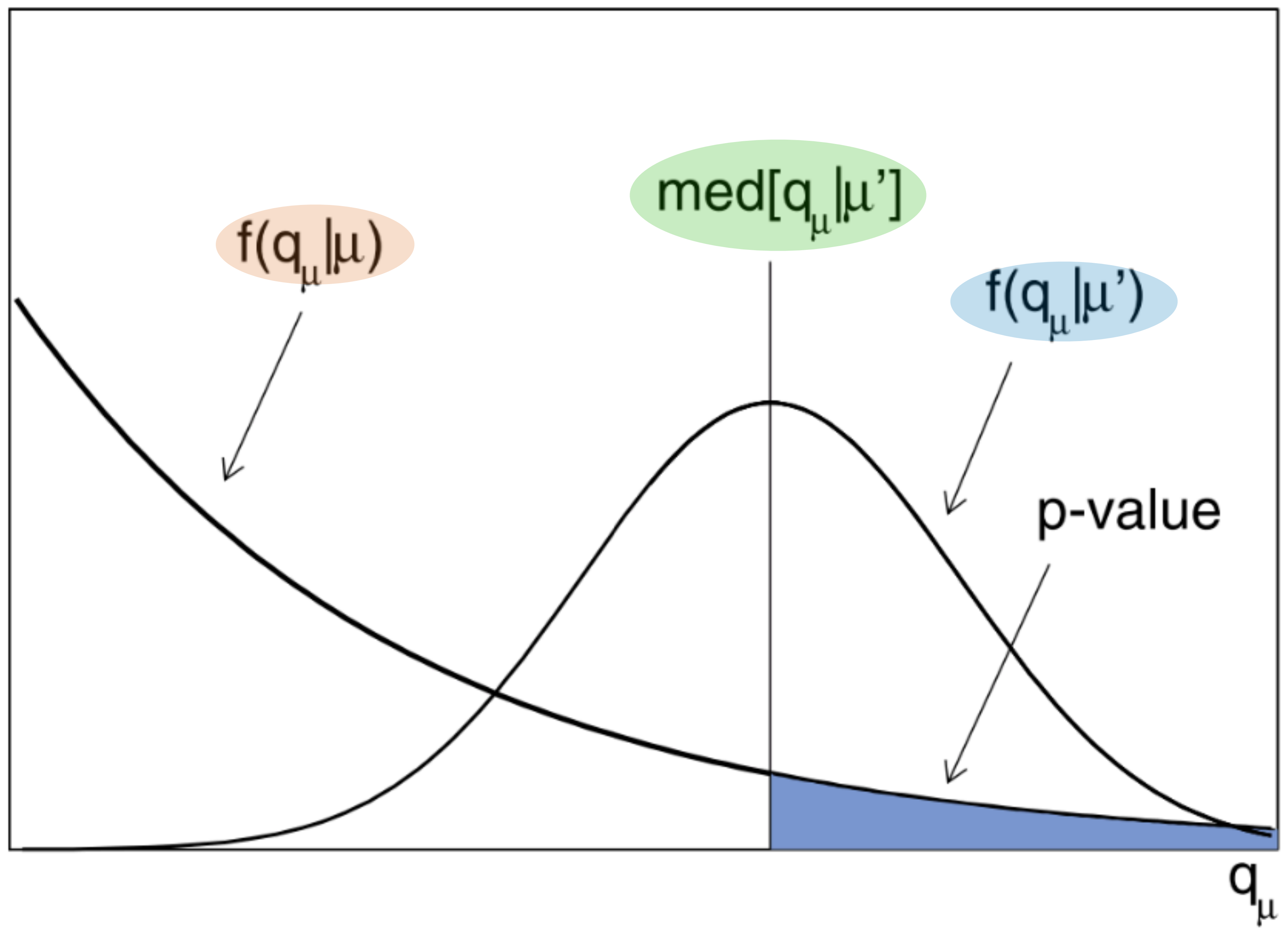


pdf for q_μ assuming a strength parameter μ

pdf for q_μ assuming a different strength parameter value μ'
shifted to higher value of q_μ, corresponding on average to lower p-values

median q_μ assuming the alternative value μ'
as the p-value is a monotonic function of q_μ, this is equal to the median p-value assuming μ'

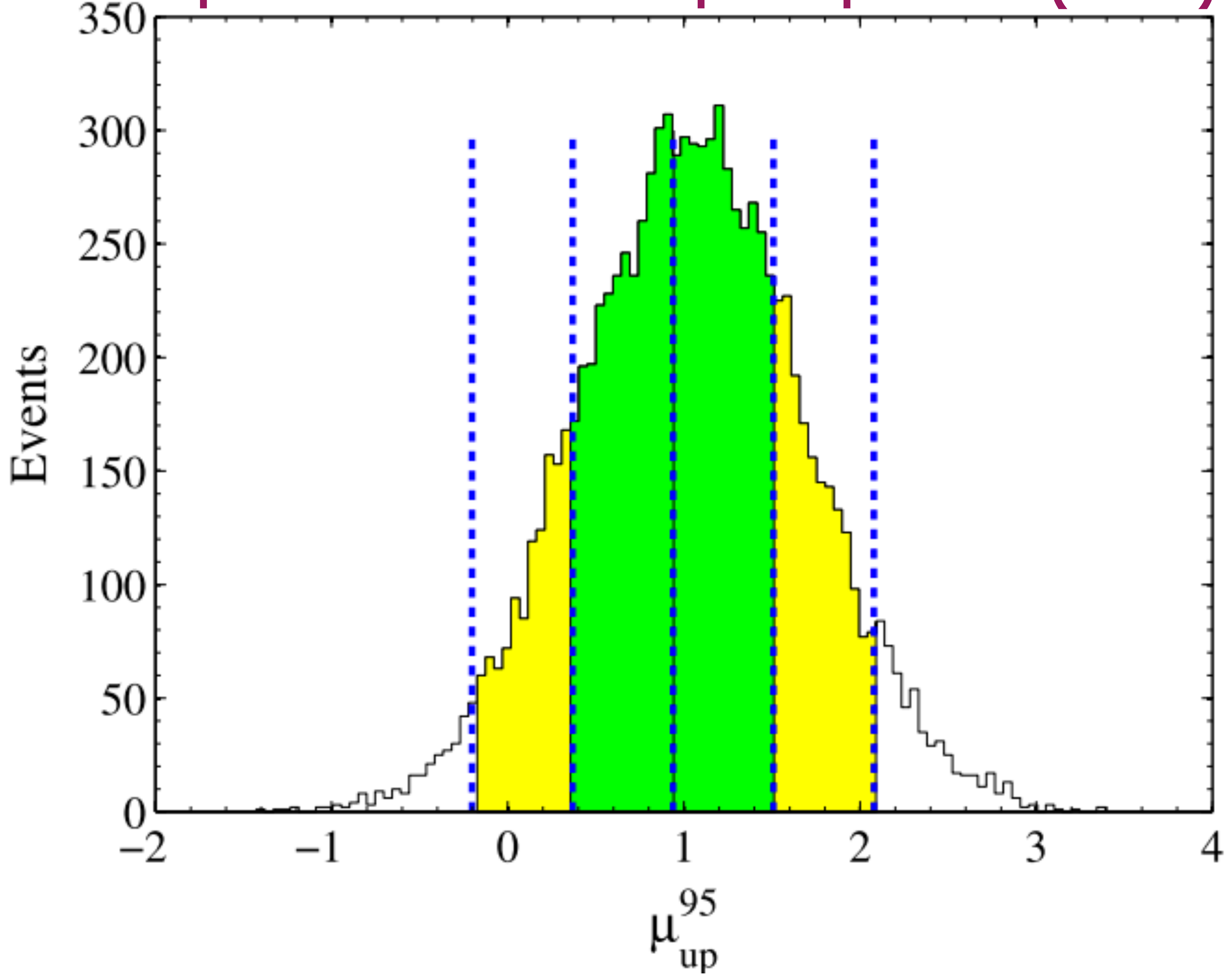
► **this characterises the sensitivity of an experiment!**

Sensitivity of an experiment

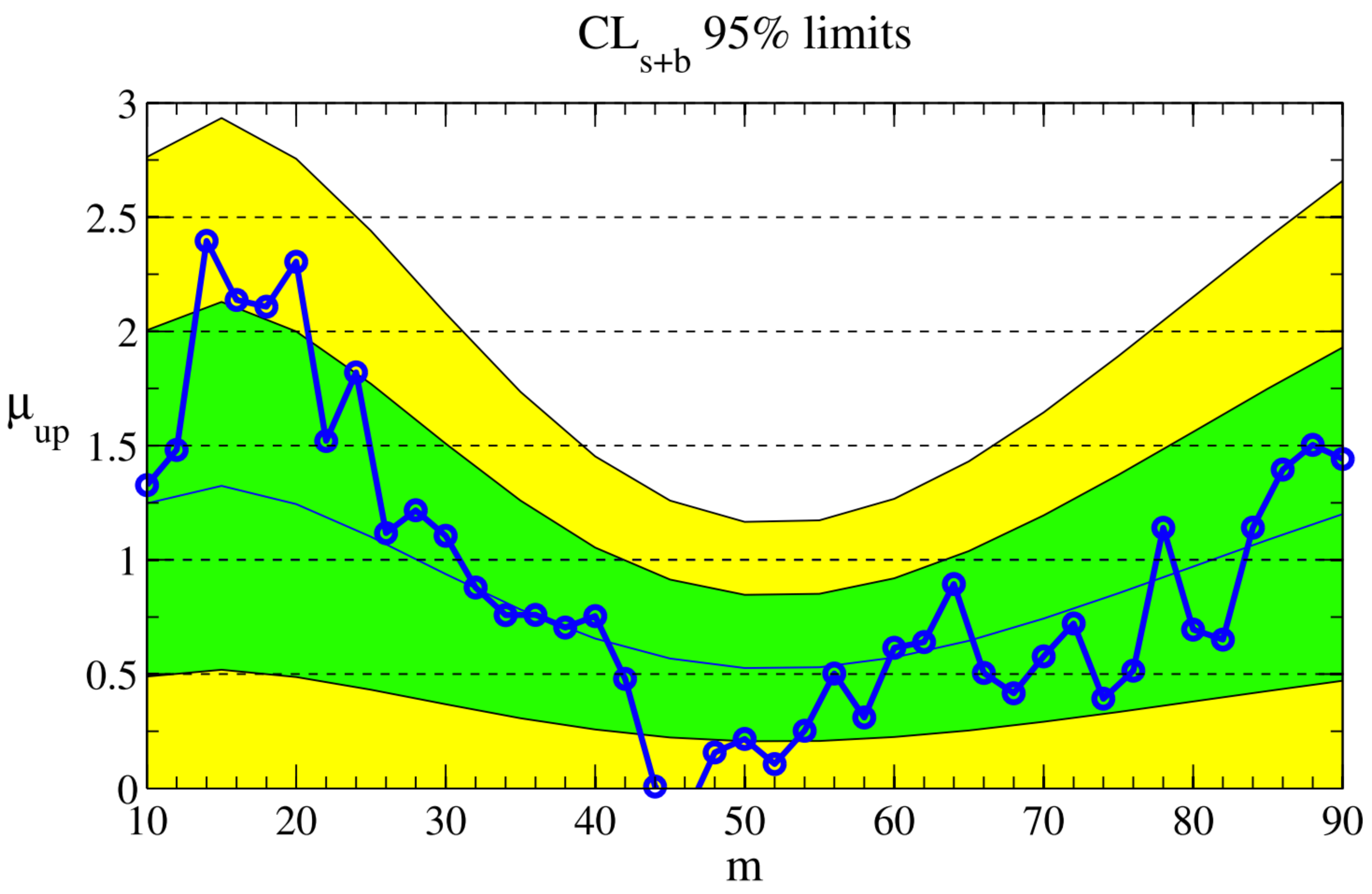
Asimov data sets

- By simulating the experiment many times with Monte Carlo, it is possible to obtain a histogram of the upper limits on μ at 95% CL;
- assuming data corresponding to the background-only hypothesis:

specific value of the peak position (mass)



search for a signal of unknown mass



The **median**, **±1σ** and **±2σ** error bands obtained from the MC pseudo-experiments.

The **vertical lines** (only left plot) are the error bands estimated directly without MC.

CL$_s$ Method

Modified Frequentist CL$_s$ method

- CL for excluding the possibility of signal on top of background (the s+b hypothesis), can be defined as:

$$\alpha_{s+b} = P_{s+b}(q_\mu \le q_{\mu,obs})$$

probability, assuming the presence of both signal and background at their hypothesised levels, that the test statistic would be less than or equal to that observed in the data

Problem

- To quote exclusion limit: confidence level $CL_{s+b} = 1 - \alpha_{s+b}$;
- if too few candidates are observed to account for the estimated background, then any signal, and even the background itself, may be excluded at a high confidence level!

CL$_s$ method is the solution!

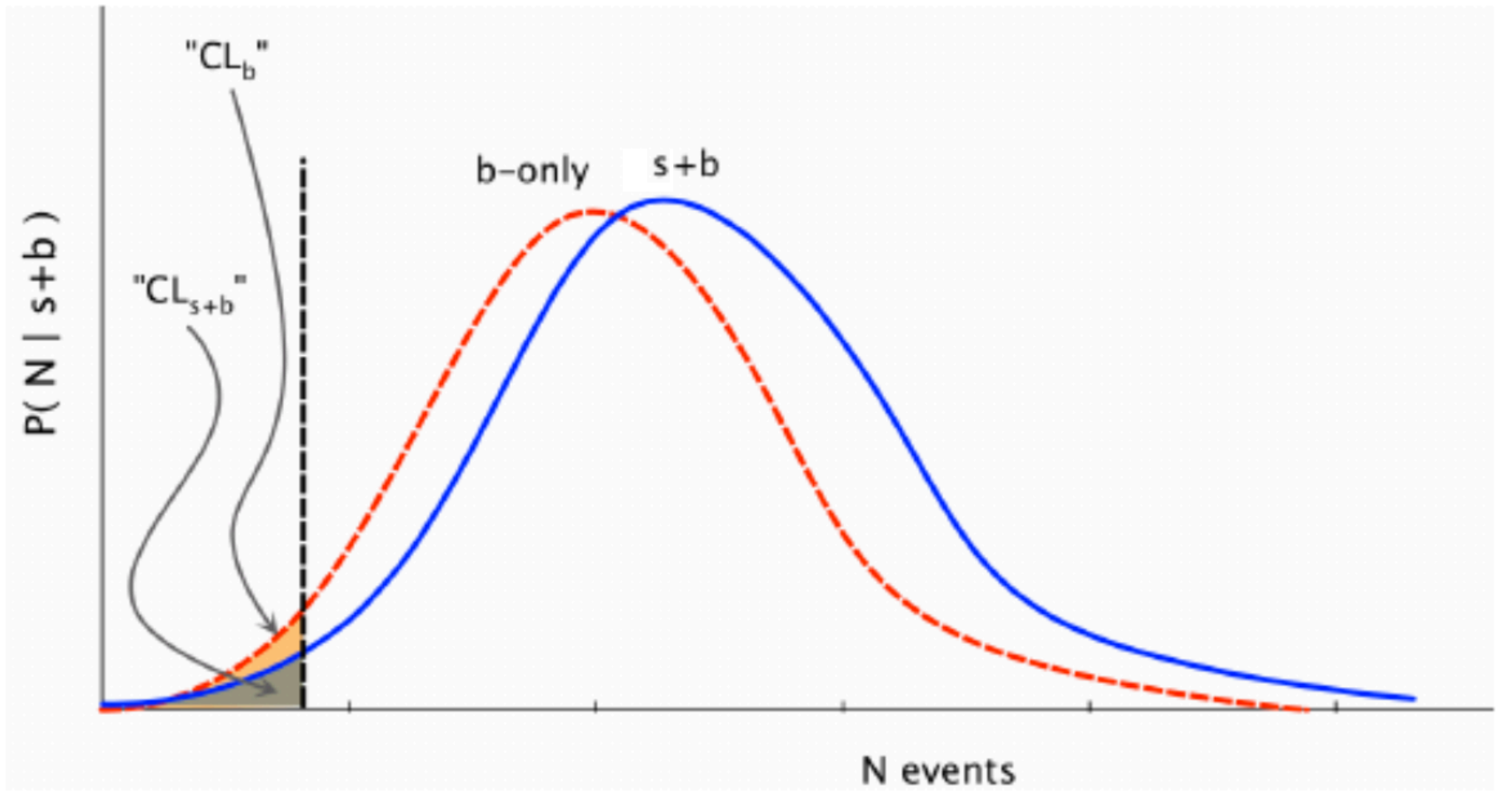
- computing the confidence level for the background alone;

$$\alpha_b = P_b(q_\mu \le q_{\mu,obs})$$

to quantify the confidence of a potential discovery, as it expresses the probability that background processes would give a number of events smaller than or equal to the number of observed candidates

- To quote exclusion limit: confidence level $CL_b = 1 - \alpha_b$;
- CLs computed as the ratio:

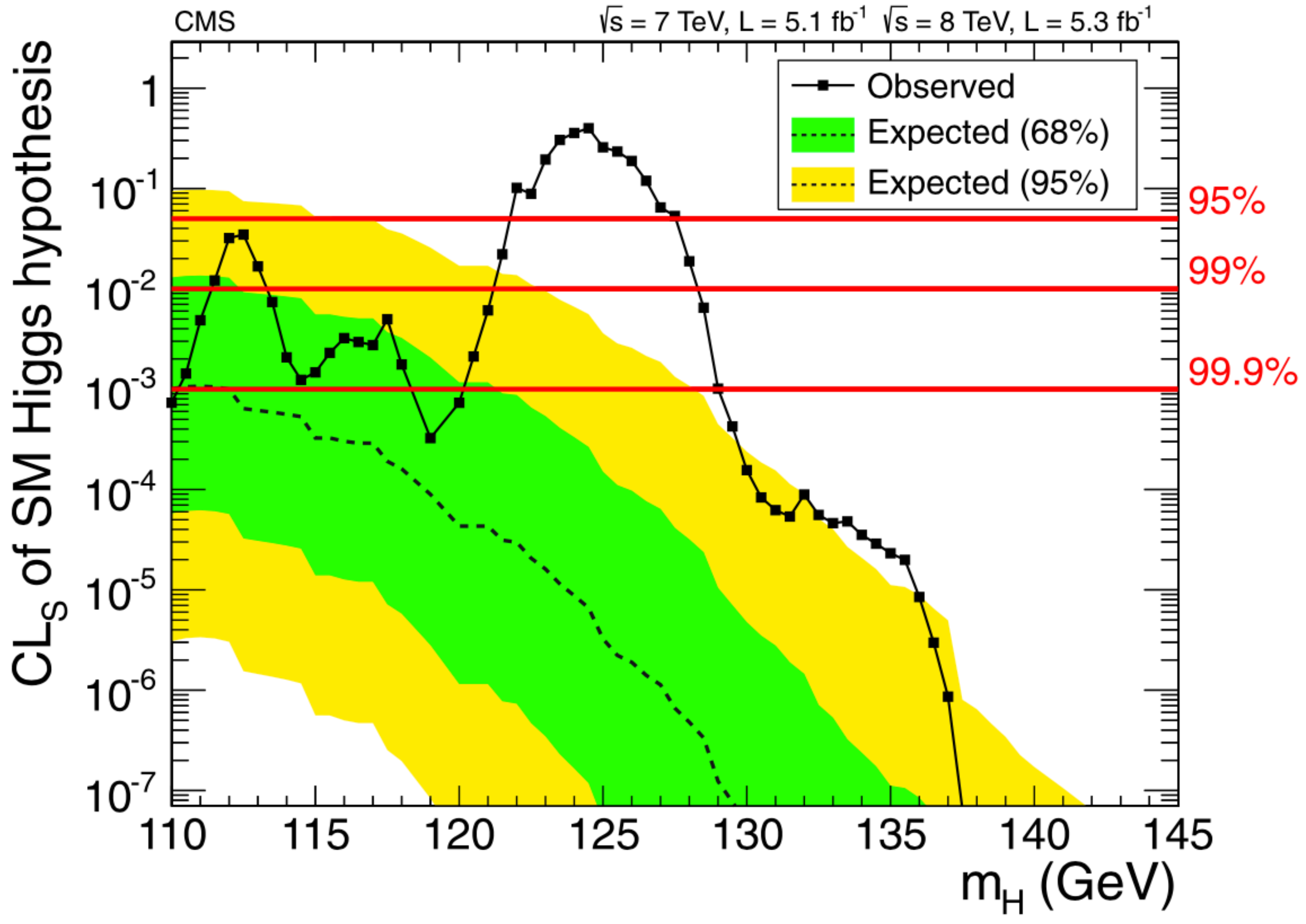
$$CL_s = CL_{s+b}/CL_b$$

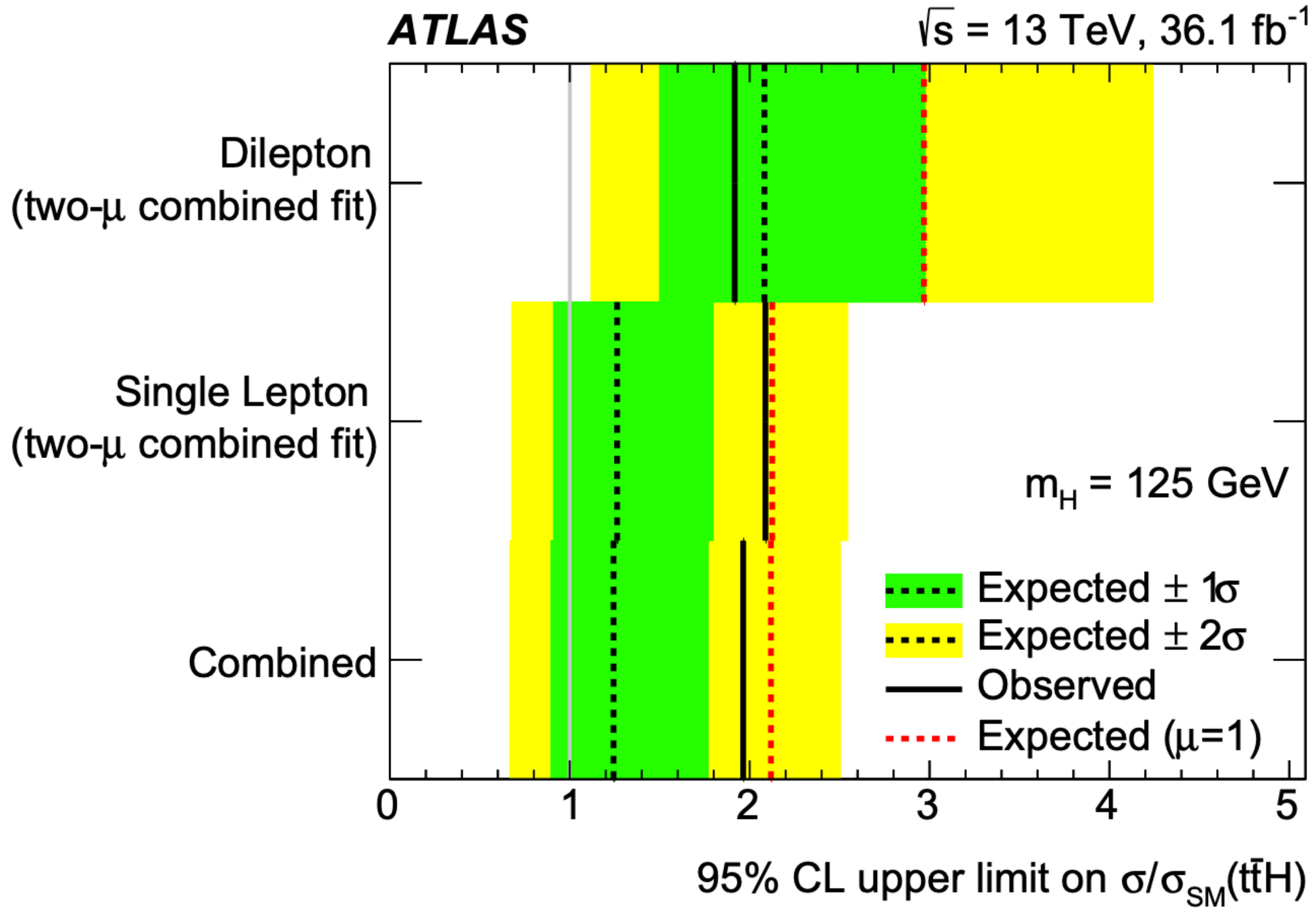


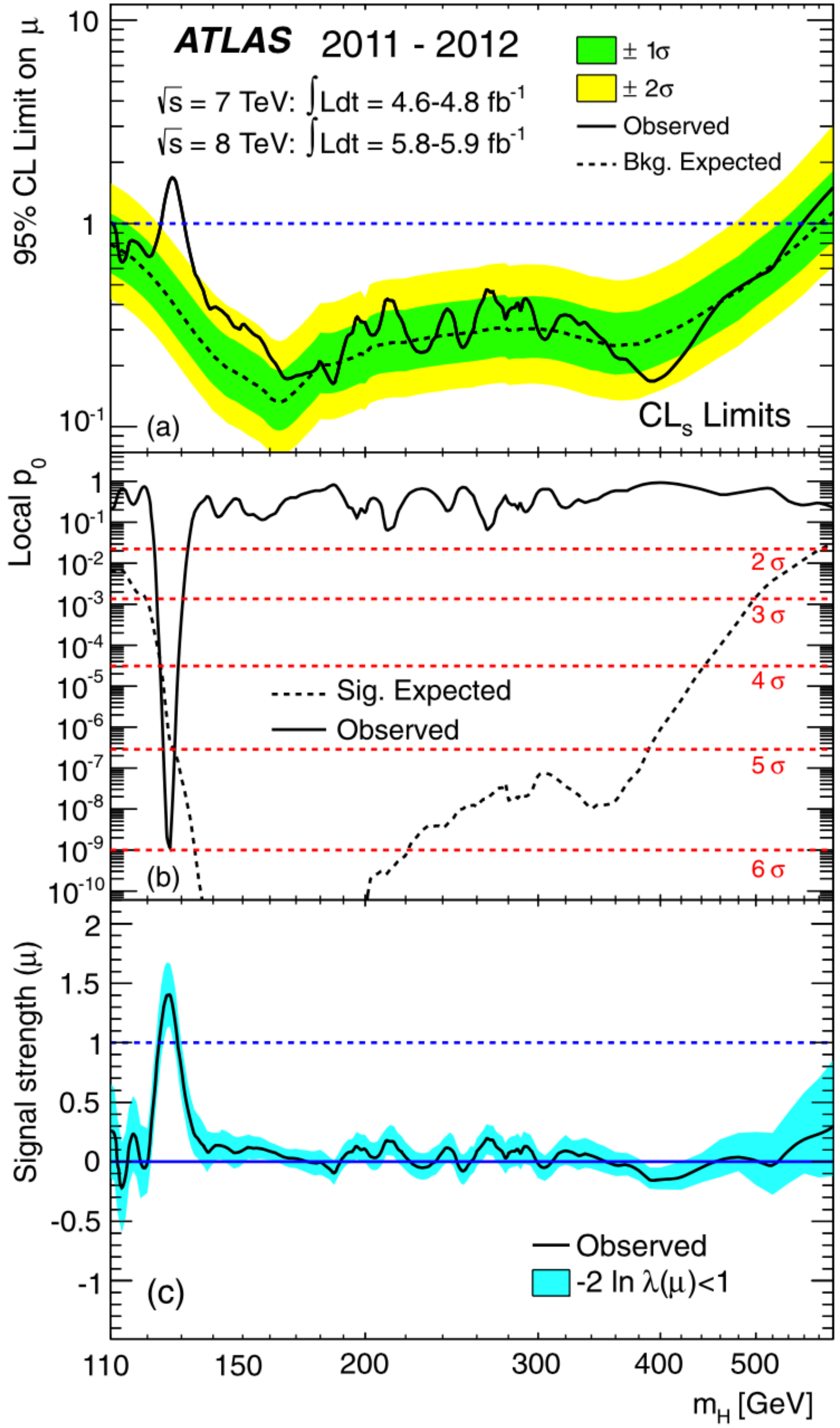
CLs Method - examples

examples

$$CL_s = CL_{s+b}/CL_b$$







Supporting material