Short Course: Dependence Models Generated via Line Integrals

Nikolai Kolev
[1]Institute of Mathematics and Statistics, University of Sao Paulo
e-mail: nkolev@ime.usp.br

1 Introduction and preliminaries

The line integral representation of the joint survival function $S(x, y)$ of a non-negative continuous random vector is known since Marshall (1975). It is given by

$$S(x, y) = \exp\left\{ - \int_{\mathcal{C}} \mathbf{R}(\mathbf{z})\, d\mathbf{z} \right\},$$

where $\mathcal{C}$ is any sufficiently smooth continuous path beginning at $(0, 0)$ and terminating at (x, y) on a open connected region in the first quadrant and $\mathbf{R}(x, y) = (r_1(x, y), r_2(x, y))$ is the hazard vector, with components

$$r_1(x, y) = \frac{\partial}{\partial x}\Big[- \ln S(x, y) \Big] \quad \text{and} \quad r_2(x, y) = \frac{\partial}{\partial y}\Big[- \ln S(x, y) \Big].$$

Surprisingly, a little attention has been given in literature to such a powerful tool for building multivariate models. A notable exception is the working paper by Dan Corro (2001), where one can find few life-insurance interpretations and applications.

Depending on the real problem at hand in continuous case, one might impose appropriate functional form of the sum $r_1(x, y) + r_2(x, y)$, indicating the most probable future behavior of the process under the absence of information. For instance, Pinto and Kolev (2015) assumed that

$$r(x, y) = r_1(x, y) + r_2(x, y) = a_0 + a_1 A_1(x) + a_2 A_2(y) \quad \text{for all} \quad x, y \geq 0, \qquad (1)$$

where a_0, a_1 and a_2 are non-negative parameters with non-negative increasing differentiable functions $A_1(x)$ and $A_2(y)$. In Pinto and Kolev (2016) is investigated the particular case when $A_1(x) = x$ and $A_2(y) = y$ in (1), leading to a huge class of bivariate models.

In Kolev (2020) the notion of a discrete line integral on uniform grids is introduced and many well known and new bivariate discrete models are generated.

2 Course outline

The main goal of the course is to show the importance of the line integral for constructing multivariate continuous and discrete models taking into account the physical nature of the process, which is governed by the components of the hazard vector. This new methodology will be illustrated by many examples. Actuarial applications involving Green's theorem will be presented.

OUTLINE:

1. Line Integral and Fundamental Theorem of Calculus;
2. Exponential Representation of Bivariate Survival Function;
3. Bivariate Continuous Case - Examples and Sibuya's Aging Property;
4. Green's Theorem Applications and Optimal Path;
5. Discrete Line Integral on Uniform Grids: Construction and Properties;
6. Exponential Representation for Bivariate Discrete Models;
7. Discussion and Further Research.

References

Dan Corro (2001). Modelling multidimensional survival with hazard vector filed. *CAS forum*, Winter

Kolev, N. (2020). Discrete line integral on uniform grids: probabilistic interpretation and applications. *Brazilian Journal of Probability and Statistics* **34**, 821-843.

Marshall, A. W. (1975). Some comments on the hazard gradient. *Stochastic Processes and their Applications* **3**, 293-300.

Pinto, J. and Kolev, N. (2015). Sibuya-type bivariate lack of memory property. *Journal of Multivariate Analysis* **134**, 119-128.

Pinto, J. and Kolev, N. (2016). A class of continuous bivariate distributions with linear sum of hazard gradient components. *Journal of Statistical Distributions and Applications* **3**, 1-17.